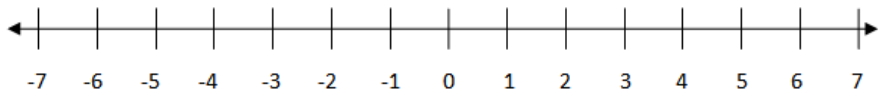
Solving Absolute Value Equations

Absolute Value Review

The absolute value of a number represents the **undirected distance** from that number to the origin on the number line, that is, the distance from x to 0.



The absolute value does not depend on whether the number is to the left or right of 0. It only depends on its distance from 0. The absolute value of a number a is written $|a|$.

(EXTRA) Examples:

| 5 | = 5 |-3| = 3 | 0 | = 0 - | -8 | = - 8

| -6 | + | 4 | =
6 + 4 = 10

| -14 | - |- 11 | =
14 - 11 = 3

Another way of thinking: Visualize yourself standing on the number line. If you were standing at 5, it would take you 5 steps to reach 0. The absolute value of 5, or |5|, is 5. If you were at -3, it would take you 3 steps to reach 0. The absolute value of -3, or |-3|, is 3. If you were at 0, it would take 0 steps to reach 0. The absolute value of 0, or |0|, is 0.

Basic Absolute Value Equations

Absolute Value Equation	Equivalent Equation	Solution Set
$\|x\| = k$ where $k > 0$	$x = k$ or $x = -k$	$\{k, -k\}$
$\|x\| = 0$	$x = 0$	$\{0\}$
$\|x\| = k$ where $k < 0$		$\varnothing$

Steps for Solving Absolute Value Equations:

1. Isolate the absolute value on one side of the equation.
2. Write the **two** equations that are equivalent to the absolute value equation.
3. Solve each equation.
4. Check each answer in the original absolute value equation.

$\lvert x \rvert = 5$	$\lvert x \rvert = -6$	$\lvert x - 7 \rvert = 3$
$x = -5$ OR $x = 5$		$x-7 = -3$ OR $x-7 = 3$
$\{-5, 5\}$	No solution or $\emptyset$	$x-7+7 = -3+7$ $\quad$ $x-7+7 = 3+7$
List solutions in numerical order.	Absolute value cannot be negative.	$x = 4$ $\quad$ $x = 10$
		$\{4, 10\}$

TRY: $\lvert x \rvert = 2$ $\qquad$ $\lvert 7 - x \rvert = 15$

$\lvert 5x + 2 \rvert = -3$ $\qquad$ $\lvert 3 - 3x \rvert = 4$

In instances where the absolute value is NOT isolated, isolate it first on one side, then solve.

DO NOT determine if the equation can be solved or not until the absolute value is by itself.

$4-3\lvert x-2\rvert=-8$ $4-4-3\lvert x-2\rvert=-8-4$ $\frac{-3\lvert x-2\rvert}{-3}=\frac{-12}{-3}$ $\lvert x-2\rvert=4$ $x-2=-4$ or $x-2=4$ $x-2+2=-4+2$ or $x-2+2=4+2$ $x=-2$ or $x=6$ $\{-2,6\}$	Subtract 4 from each side to isolate the term with the absolute value. Divide both sides by -3 to isolate the absolute value. Solve as before.

TRY: 5 - 4| x+3 | = -43 (EXTRA) | x+3 |- 5 = 7 Did you get {-15,9}?

Double Absolute Value

Treat this one as if the right side was not an absolute value. | w – 6 | = | 3 – 2w |

Try:

Think of it as | w – 6 | = 3 – 2w

Then set it up as: w – 6 = + (3 – 2w) or w – 6 = – (3 – 2w) and solve each part