Linear Inequalities: Addition and Multiplication Properties

Addition Property of Inequality: If the same number is added to both sides of an inequality, then the solution set to the inequality is unchanged. For any algebraic expressions A, B, and C, if A < B, then A + C < B + C

[Note: No change in direction of inequality symbol]

$x-5<8$
$x-5+5<8+5$
$x<13$
$\{x \mid x<13\}$ Interval Notation: $(-\infty,13)$
$7-x\le 9$
$7-x+x\le 9+x$
$7\le 9+x$
$7-9\le 9-9+x$
$-2\le x$
$\{x \mid -2\le x\}$ or $\{x \mid x\ge -2\}$ Either way, the x is greater than or equal to -2.
$[-2,\infty)$ The bracket indicates 'or equal to' -2.

TRY: $x+6<15$ $5+x\ge 10$

Multiplication Property of Inequality: If both sides of an inequality are multiplied by the same **positive** number, then the solution set to the inequality is unchanged. For any algebraic expressions A, B, and C, if A < B and C > 0 (positive), then A • C < B • C

[Note: No change in direction of inequality symbol]

Examples:

$\frac{1}{2}x\le 10$
$2\cdot\frac{1}{2}x\le 10\cdot 2$
$x\le 20$
$\{x \mid x\le 20\}$
$(-\infty,20]$

$5x>15$
$\frac{1}{5}\cdot 5x>15\cdot\frac{1}{5}$ also thought of as $\frac{5x}{5}>\frac{15}{5}$
$x>3$
$\{x \mid x>3\}$
$(3,\infty)$

TRY: $\frac{1}{4}x \geq 8$ $4x < 8$

Multiplication Property of Inequality: If both sides of an inequality are multiplied by the same **negative** number and the <u>inequality symbol is reversed</u>, then the solution set to the inequality is unchanged. For any algebraic expressions A, B, and C, if A < B and C < 0 (negative), then A • C > B • C

[Note: Direction of inequality symbol is reversed]

	$-x > 3$	$-5x \geq 35$	$8 > -\frac{x}{4}$
	$-x \cdot (-1) > 3 \cdot (-1)$	$-5x \cdot \left(-\frac{1}{5}\right) \geq 35 \cdot \left(-\frac{1}{5}\right)$	$8 \cdot (-4) > -\frac{x}{4} \cdot (-4)$
SIGN REVERSED	$x < -3$	$x \leq -7$	$-32 < x$
	$\{x \mid x < -3\}$	$\{x \mid x \leq -7\}$	$\{x \mid x > -32\}$ putting x first
	$(-\infty, -3)$	$(-\infty, -7]$	$(-32, \infty)$

TRY: $-\frac{1}{3}x < 9$ $-8x \leq 16$

One can multiply and divide both sides of an inequality by the same positive expression and maintain the direction of the inequality symbol. But, <u>if one multiplies or divides both sides of an inequality by the same negative expression, the direction of the inequality symbol must be reversed</u>.

Solve each of the following inequalities. Express the solution set in interval notation and graph it.

$3x-2<6$

$3x-2+2<6+2$

$3x<8$

$\frac{3x}{3}<\frac{8}{3}$

$x<\frac{8}{3}$

$\{x \mid x<\frac{8}{3}\}$

$(-\infty,\frac{8}{3})$

$19\le 5-4x$

$19-5\le 5-5-4x$

$14\le -4x$

$\frac{14}{-4}\le\frac{-4x}{-4}$

Be sure to FLIP the sign!

$-\frac{7}{2}\ge x$

$x\le -\frac{7}{2}$

$\{x \mid x\le -\frac{7}{2}\}$

$(-\infty,-\frac{7}{2}]$

$2x+3>2(x-4)$

$2x+3>2x-8$

$2x+3-3>2x-8-3$

$2x>2x-11$

$2x-2x>2x-2x-11$

$0>-11$

This is a TRUE statement. Therefore, the entire number line is the solution.

$\{x \mid x\in \text{Re}als\}$

$(-\infty,\infty)$

TRY: 3x – 6 <15 $7\le 5-3x$ 3x + 2 > -5+ 3x