Lesson 21: Quadratic Equations

Review of Quadratic Equations

Review of the Definition of a Quadratic Equation in One Variable

A quadratic equation in one variable is any second-degree equation that can be written in the form $ax^2 + bx + c = 0$ where a, b, and c are real numbers and $a \neq 0$.

If the equation is in this form, we call it the **standard form** of a quadratic equation in one variable, x.

[a cannot be zero. If a were zero then the x^2 term would be zero and one would have a first-degree (linear) equation, not a quadratic equation.]

Review of the Zero Product Property

Given real numbers p and q, if $pq = 0$, then $p = 0$ or $q = 0$.

Review of the steps for Solving a Quadratic Equation by Factoring

1. Write the quadratic equation in standard form ($ax^2 + bx + c = 0$) with the leading coefficient *positive*.
   If the first term is negative, multiply every term of the equation by -1 to **make it positive**.
2. Completely factor the quadratic expression.
3. Use the zero factor property to set each of the factors containing the variable equal to 0.
4. Solve the simpler linear equations.
5. Check the solution(s) in the original equation.

Examples:

$$
\begin{aligned}
x^2 + 6x + 8 &= 0 \\
(x+4)(x+2) &= 0 \\
x + 4 = 0 \text{ or } x + 2 &= 0 \\
x = -4 \text{ or } x &= -2 \\
\{-4, -2\}
\end{aligned}
$$

$$
\begin{aligned}
w^3 - 25w &= 0 \\
w(w-5)(w+5) &= 0 \\
w = 0 \text{ or } w - 5 = 0 \text{ or } w + 5 &= 0 \\
w = 0 \text{ or } w = 5 \text{ or } w &= -5 \\
\{-5, 0, 5\}
\end{aligned}
$$

TRY:

$x^2 - 11x + 18 = 0$

$8x^3 + 4x^2 - 8x - 4 = 0$ Solve by grouping, factor completely.

Square Root Property

What number times itself is 9? 3 Is there another number that also works? -3

So in looking at $x^2 = 9$, either 3 or -3, when substituted in place of x, would make the statement true. In solving quadratic equations, we now need to consider both the positive and the negative results.

Square Root Property (sometimes called the Even Root Property)

When *n* is a positive even integer,

if $k > 0$, then $x^n = k$ is equivalent to $x = \pm\sqrt[n]{k}$. $x^2 = 4$ is equivalent to $x = \pm 2$

if $k = 0$, then $x^n = k$ is equivalent to 0. $x^2 = 0$ is equivalent to $x = 0$

if $k < 0$, then $x^n = k$ has no real solution. $x^2 = -4$ has no real solution

Examples:
Solve for the unknown using the Square Root Property

$$x^2 = \frac{9}{4}$$
$$\sqrt{x^2} = \sqrt{\frac{9}{4}}$$
$$x = \pm\frac{3}{2}$$
$$\left\{-\frac{3}{2}, \frac{3}{2}\right\}$$

$$a^2 = 32$$
$$\sqrt{a^2} = \sqrt{32}$$
$$a = \pm 4\sqrt{2}$$
$$\{-4\sqrt{2}, 4\sqrt{2}\}$$

$$x^2 = 25$$
$$\sqrt{x^2} = \sqrt{25}$$
$$x = \pm 5$$
$$\{-5, 5\}$$

TRY:

$$x^2 = \frac{16}{25}$$

$$x^2 = 98$$

Would the same process work to solve for (x-4):

$$(x-4)^2 = 25$$

$$\sqrt{(x-4)^2} = \sqrt{25}$$

$$(x-4) = \pm 5$$

Now solve for x. $x-4=5$ or $x-4=-5$

$x=9$ or $x=-1$

$\{-1, 9\}$

The process of using the square root property to solve certain forms of quadratic equations is called **extracting the roots**.

$$(a-3)^2 = 8$$

$$\sqrt{(a-3)^2} = \sqrt{8}$$

$$a-3 = \pm 2\sqrt{2}$$

$a-3 = +2\sqrt{2}$ or $a-3 = -2\sqrt{2}$

$a = 3+2\sqrt{2}$ or $a = 3-2\sqrt{2}$

$\{3-2\sqrt{2}, 3+2\sqrt{2}\}$

$$(2m-7)^2 = 12$$

$$\sqrt{(2m-7)^2} = \sqrt{12}$$

Remember: $\sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$

$$2m-7 = \pm 2\sqrt{3}$$

$$2m = 7 \pm 2\sqrt{3}$$

$$m = \frac{7 \pm 2\sqrt{3}}{2}$$

$$\left\{\frac{7-2\sqrt{3}}{2}, \frac{7+2\sqrt{3}}{2}\right\}$$

Do not be tempted to remove the 2's in this answer. In order to reduce by 2, a factor of 2 must be able to be factored out from each of the two terms in the numerator. There is not a factor of 2 in the number 7, so one cannot factor out a 2. Therefore, one cannot reduce the fraction by 2.

TRY:

$(x-5)^2 = 9$

$(2x+7)^2 = 15$

Using the Square Root property with Complex Numbers

$x^2 = -49$ $\sqrt{x^2} = \sqrt{-49}$ $x = \pm\sqrt{-49}$ $x = \pm\sqrt{-1 \cdot 49}$ $x = \pm 7i$ The solution set is $\{-7i, 7i\}$	$(k+5)^2 + 13 = 4$ $(k+5)^2 = 4 - 13$ $(k+5)^2 = -9$ $\sqrt{(k+5)^2} = \sqrt{-9}$ $k + 5 = \pm 3i$ $k = -5 \pm 3i$ The solution set is $\{-5-3i, -5+3i\}$	$(5m-7)^2 + 20 = 0$ $(5m-7)^2 = -20$ $\sqrt{(5m-7)^2} = \sqrt{-20}$ $5m - 7 = \pm\sqrt{-20}$ $5m - 7 = \pm i\sqrt{4 \cdot 5}$ $5m - 7 = \pm 2i\sqrt{5}$ $5m = 7 \pm 2i\sqrt{5}$ $m = \frac{7 \pm 2i\sqrt{5}}{5}$ The solution set is $\left\{\frac{7-2i\sqrt{5}}{5}, \frac{7+2i\sqrt{5}}{5}\right\}$

TRY:

$x^2 = -36$

$(k-3)^2 + 12 = 4$

$(2m+3)^2 + 14 = 6$

Using a Radical in a Formula

The portion of the line between the two points is called a **line segment**.

A line has infinite length, while a line segment has a specific length.

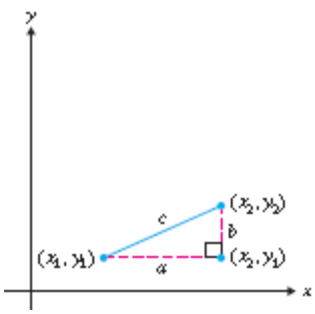
The formula for finding the distance between two points (the length of a line segment) includes a radical.

Distance Formula (based on the Pythagorean Theorem)

The distance between any two points (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

[In my words – the distance between any two points is the square root of …
the 'run along the x-axis' squared + the 'rise along the y-axis' squared]



The distance formula is derived from the Pythagorean Theorem:
$a^2 + b^2 = c^2$

Using the square root property to solve for c, the result is:

$c = \sqrt{a^2 + b^2}$ or $c = \sqrt{(leg1)^2 + (leg2)^2}$

In the diagram, one can see that 'a' or 'leg1' represents the change in x values.
'b' or 'leg2' represents the change in x values.

Find the distance between the points (4,2) and (12,8). [length of 'c' in the Pythagorean Theorem]

$d = \sqrt{(12-4)^2 + (8-2)^2}$ $\quad$ $d = \sqrt{(8)^2 + (6)^2}$ = $\sqrt{64+36}$ = $\sqrt{100}$ = ± 10

Since the problem involves distance, + 10 is the reasonable solution.
The distance between the two points is 10 units.

TRY:

Find the distance between (2, 5) and (6, 8)

Completing the Square

Think about $x^2+8x+16$.

This can be written $(x+4)^2$.

The expression $x^2+8x+16$ is a perfect square trinomial.

Perfect square trinomials: $a^2-2ab+b^2=(a-b)^2$ and $a^2+2ab+b^2=(a+b)^2$

Necessary Conditions for a Perfect Square Trinomial

4. The first term must have a positive coefficient and be a perfect square, a^2.
5. The last term must have a positive coefficient and be a perfect square, b^2.
6. The middle term must be twice the product of the bases of the first and last terms, $2ab$ or $-2ab$.

Is $x^2+16x+64$ a Perfect Square Trinomial?

YES – since it can be factored: $(x+8)^2$

Perfect square trinomials are created when a binomial is squared.

Think about the following: $x^2+6x+?$

What number could be used for the $?$ to create a perfect square trinomial?

$$x^2+6x+\underline{9} \quad \text{or} \quad (x+3)^2$$

Complete the following perfect square trinomial: $(x+\_)^2 = x^2+10x+?$

5 25

Now think of the process in reverse. What values make these into perfect square trinomials?

$x^2+14x+\_\_ = (x+\_\_)^2$

49 7

$x^2-12x+\_\_ = (x-\_\_)^2$

36 6

What process will always work for $ax^2+bx+c=0$ when a=1?

x^2+14x+ ___ $(x+$ ___ $)^2$	$x^2+14x+\underline{49}$ $\downarrow \; \downarrow \tfrac{1}{2}$ $(x+\underline{7})^{2}$	1. Be sure the sign of the b term in the trinomial is the same as the sign in the binomial. 2. Take ½ of the coefficient of the b term of the trinomial and place it in the blank of the binomial. 3. Square the last term of the binomial and use the result for the c term of the trinomial.
$x^2-12x+\underline{36}$ $\downarrow \; \downarrow \tfrac{1}{2}$ $(x-\underline{6})^2$	$x^2-5x+\underline{\tfrac{25}{4}}$ $\downarrow \; \downarrow$ $(x-\underline{\tfrac{5}{2}})^2$	$x^2+\frac{6}{7}x+\underline{\tfrac{9}{49}}$ $\downarrow \; \downarrow (\tfrac{1}{2})$ $(x+\underline{\tfrac{3}{7}})^2$ Even if the b is a fraction, still take ½ of it.

Completing the Square

The process of transforming the quadratic equation $ax^2+bx+c=0$ into the form $(x+q)^2=k$ where q and k are constants is called **completing the square**.

If one knows the first and second terms, one can find the last term to make a perfect square trinomial. It is *the square of* ½ of the coefficient of the middle term.

TRY:

$m^2 + 14m +$ ___ $\qquad (m +$ ___ $)^2$

$w^2 - 5w +$ ___ $\qquad (w -$ ___ $)^2$

$p^2 + \frac{6}{5}p +$ ___ $\qquad (p +$ ___ $)^2$

Do you see how easily these perfect square trinomials can be factored? TRY THESE.

$y^2-10y+25$ $\qquad$ $w^2+w+\frac{1}{4}$ $\qquad$ $m^2-\frac{6}{5}m+\frac{9}{25}$

$(y-5)^2$ $\qquad$ $\left(w+\frac{1}{2}\right)^2$ $\qquad$ $\left(m-\frac{3}{5}\right)^2$

Solving Quadratics by Completing the Square

By blending the concept of Completing the Square and the Square Root property, quadratic equations that are not easily factored can now be easily solved.

Solving by Completing the Square $ax^2 + bx + c = 0$

1. If $a = 1$, proceed to step 2. If $a \neq 1$, divide each term of the equation by a before completing the square.

2. Write the equation with the variable terms on one side and the constant on the other. That is, move c to the other side.
3. Determine $\frac{1}{2}$ of the b value. Square that and add it to each side of the equation.
4. Write the side with the terms containing the variable as a perfect square $\left(x - \frac{1}{2}b\right) or \left(x + \frac{1}{2}b\right)$ and combine terms on the other side.

5. Extract the roots using the square root property and solve the resulting linear equations.

Examples – Follow the 5 steps given above.

$x^2 - 6x - 7 = 0$ $x^2 - 6x + \_\_ = 7 + \_\_$ Move the -7 to the right side. $\frac{1}{2}(b) = \frac{1}{2}(6) = 3$ and $3^2 = 9$ so add 9 to both sides. $x^2 - 6x + 9 = 7 + 9$ $(x-3)^2 = 16$ $\sqrt{(x-3)^2} = \sqrt{16}$ Now use the square root property. $x - 3 = \pm 4$ $x = 3 + 4$ or $x = 3 - 4$ $x = 7$ or $x = -1$ $\{-1, 7\}$	$3x^2 - 6x - 18 = 0$ Divide first by 3 to make $a = 1$. $x^2 - 2x - 6 = 0$ $x^2 - 2x + \_\_ = 6 + \_\_$ Move the 6 to the right side. $\frac{1}{2}(b) = \frac{1}{2}(2) = 1$ and $1^2 = 1$ so add 1 to both sides. $x^2 - 2x + 1 = 6 + 1$ $(x-1)^2 = 7$ $\sqrt{(x-1)^2} = \sqrt{7}$ Use the square root property. $x - 1 = \pm\sqrt{7}$ $x = 1 \pm \sqrt{7}$ $\{1-\sqrt{7}, 1+\sqrt{7}\}$

TRY:

Solve by completing the square:

$y^2 - 3y - 10 = 0$ $\qquad$ $2m^2 - 8m - 20 = 0$

Sometimes …

…the solutions are not pretty.	…the equations are not pretty.	…the solutions are not real.
$4x^2 + 4x - 2 = 0$ Divide by 4. $x^2 + x - \frac{1}{2} = 0$ $x^2 + x + \_\_ = \frac{1}{2} + \_\_$ $x^2 + x + \frac{1}{4} = \frac{1}{2} + \frac{1}{4}$ $(x + \frac{1}{2})^2 = \frac{3}{4}$ $\sqrt{(x + \frac{1}{2})^2} = \sqrt{\frac{3}{4}}$ $x + \frac{1}{2} = \pm\sqrt{\frac{3}{4}}$ $x + \frac{1}{2} = \pm\frac{\sqrt{3}}{2}$ $x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}$ $x = \frac{-1 \pm \sqrt{3}}{2}$ $\left\{\frac{-1-\sqrt{3}}{2}, \frac{-1+\sqrt{3}}{2}\right\}$	Put in proper form first. $2s(2s+5) - 20 = 18s$ $4s^2 + 10s - 20 - 18s = 0$ $4s^2 - 8s - 20 = 0$ Divide by 4. $s^2 - 2s - 5 = 0$ $s^2 - 2s + \_\_ = 5 + \_\_$ $s^2 - 2s + 1 = 5 + 1$ $(s-1)^2 = 6$ $\sqrt{(s-1)^2} = \sqrt{6}$ $s - 1 = \pm\sqrt{6}$ $s = 1 \pm \sqrt{6}$ $\{1-\sqrt{6}, 1+\sqrt{6}\}$	$x^2 - 4x + 12 = 0$ $x^2 - 4x + \_\_ = -12 + \_\_$ $x^2 - 4x + 4 = -12 + 4$ $(x-2)^2 = -8$ $\sqrt{(x-2)^2} = \sqrt{-8}$ $x - 2 = \pm\sqrt{-8}$ $x = 2 \pm \sqrt{-8}$ $x = 2 \pm i\sqrt{8}$ $\quad$ $\sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$ $x = 2 \pm 2i\sqrt{2}$ $\{2 - 2i\sqrt{2}, 2 + 2i\sqrt{2}\}$

What if we had a ‘magical’ formula that was quicker and easier to use and would always work?

What if we could develop a formula that solve for x in the generic quadratic equation $ax^2+bx+c=0$?

Developing the Quadratic Formula

Instructor’s note: Please listen to the lesson covering the following development of the quadratic formula. This is my favorite part of the entire course!

*Given the standard form of the Quadratic Equation – one can solve for x by completing the square. The resulting formula, known as the **Quadratic Formula**, will work to solve for any x.*

Given: $ax^2+bx+c=0$ where a, b, and c are real numbers and $a \neq 0$

Use the five step involving completing the square and the square root property to solve for x.

1) Make the leading coefficient 1. Divide each term by a. $\frac{ax^2}{a}+\frac{bx}{a}+\frac{c}{a}=0$

2) Rewrite the equation: $x^2+\frac{b}{a}x+\frac{c}{a}=0$

3) Complete the Square: $x^2+\frac{b}{a}x+$______ = $-\frac{c}{a}+$______ Note: ½ of $\frac{b}{a}$ is $\frac{b}{2a}$.

$$\left(\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}$$

$$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=-\frac{c}{a}+\frac{b^2}{4a^2}$$ $\frac{b^2}{4a^2}$ was added to both sides.

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$ A common denominator $4a^2$ was found and the two terms on the right side combined.

4) Take the square root: $\sqrt{\left(x+\frac{b}{2a}\right)^2}=\sqrt{\frac{b^2-4ac}{4a^2}}$ ➔ $\left(x+\frac{b}{2a}\right)=\frac{\pm\sqrt{b^2-4ac}}{2a}$

5) Isolate the x: $x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$ and combine the right side terms: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

YEAH!!! This is a wonderful formula that solves for x in any quadratic equation! KEEP IT HANDY!

Using the Quadratic Formula

Quadratic Formula

If $ax^2+bx+c=0$ and $a\neq 0$, then $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

where a is the coefficient of the second-degree term, b is the coefficient of the first-degree term, and c is the constant.

To Solve Quadratic Equations Using the Quadratic Formula

1. Write the quadratic equation in **standard form** with a leading *positive* coefficient.
2. Identify the values of a, b, and c.
3. Substitute these values into the quadratic formula.
4. Simplify the resulting expression.

Keeping the formula handy…. $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$4+20x=-25x^2$	$\frac{1}{4}p^2+\frac{3}{2}p+3=0$ Mult. By 4	$-4x^2+12x-9=0$ Multiply by = -1
$25x^2+20x+4=0$ Standard form.	$p^2+6p+12=0$ Standard form.	$4x^2-12x+9=0$ Standard Form.
a = 25 b = 20 c = 4	a = 1 b = 6 c = 12	a = 4 b = -12 c = 9
$x=\frac{-(20)\pm\sqrt{(20)^2-4(25)(4)}}{2(25)}$	$p=\frac{-(6)\pm\sqrt{(6)^2-4(1)(12)}}{2(1)}$	$x=\frac{-(-12)\pm\sqrt{(-12)^2-4(4)(9)}}{2(4)}$
$x=\frac{-20\pm\sqrt{400-200}}{50}$	$p=\frac{-6\pm\sqrt{36-48}}{2}$	$x=\frac{12\pm\sqrt{144-144}}{8}$
$x=\frac{-20\pm\sqrt{200}}{50}$	$p=\frac{-6\pm\sqrt{-12}}{2}$ Bring out i.	$x=\frac{12\pm\sqrt{0}}{8}$
$x=\frac{-20\pm10\sqrt{2}}{50}$ Factor out a 10.	$p=\frac{-6\pm2i\sqrt{3}}{2}$ Factor out a 2.	$x=\frac{12}{8}$
$x=\frac{10(-2\pm\sqrt{2})}{10\cdot5}$	$p=\frac{2(-3\pm i\sqrt{3})}{2}$	$x=\frac{3}{2}$
$x=\frac{-2\pm\sqrt{2}}{5}$	$p=-3\pm i\sqrt{3}$	$\left\{\frac{3}{2}\right\}$
$\left\{\frac{-2-\sqrt{2}}{5},\frac{-2+\sqrt{2}}{5}\right\}$	$\{-3-i\sqrt{3},-3+i\sqrt{3}\}$	

TRY:

$3z^2 - 8z + 2 = 0$ $\qquad$ $-8q^2 - 2q + 1 = 0$ $\qquad$ $2y^2 + 1 = 2y$

Sometimes one method is easier than another for solving a quadratic equation.

Solving a Quadratic Equation – in General

1. Write the quadratic equation in <u>standard form</u> with a leading *positive* coefficient.

2. <u>Clear fractions</u> if necessary.

3. Solve using an appropriate method:

    a. Check to see if the polynomial can be factored. If so – **<u>solve by factoring</u>**.

    b. If $b = 0$ or the quadratic equation has the form $(px + q)^2 = k$, **<u>solve by extracting the roots</u>**, using the even-root property.

    c. **<u>Solve by using the quadratic formula</u>**. (One can also use completing the square, but using the quadratic formula is usually faster.)

Once the ‘a’, ‘b’, and ‘c’ values have been identified, one can determine how many solutions and what type (real or imaginary) of solution one should have to the problem.

Number of Solutions to a Quadratic Equation

If a, b, and c are rational numbers and $-\frac{b}{2a}$ exists, then when the **discriminant** b^2-4ac is :

<table>
<tr>
<td>Zero $b^2-4ac=0$

One rational repeated solution, namely $-\frac{b}{2a}$
(trinomial will factor)</td>
<td>Positive $b^2-4ac>0$
If b^2-4ac...
a. is a perfect square, two distinct rational solutions (will factor)
b. is not a perfect square, two distinct irrational solutions (won't factor) It is prime.</td>
<td>Negative $b^2-4ac<0$

Two distinct complex (nonreal or imaginary) solutions (won't factor) It is prime.</td>
</tr>
<tr>
<td>$4x^2-12x+9=0$

a = 4 b = -12 c = 9

$b^2-4ac=0$

$(-12)^2-4(4)(9)=144-144=0$
One rational solution: $\left\{\frac{3}{2}\right\}$</td>
<td>a. $-8q^2-2q+1=0$
$8q^2+2q-1=0$
a = 8 b = 2 c = -1
$b^2-4ac>0$
$(2)^2-4(8)(-1)=4+32=36$
36 > 0 and 36 is a perfect square
Two rational solutions: $\left\{-\frac{1}{2},\frac{1}{4}\right\}$
b. $3z^2-8z+2=0$
a = 3 b = -8 c = 2
$b^2-4ac>0$
$(-8)^2-4(3)(2)=64-24=40$
40 > 0, but 40 is not a perfect square
Two irrational solutions:
$\left\{\frac{4-\sqrt{10}}{3},\frac{4+\sqrt{10}}{3}\right\}$</td>
<td>$2y^2+1=2y$
$2y^2-2y+1=0$
a = 2 b = -2 c = 1
$b^2-4ac<0$

$(-2)^2-4(2)(1)=4-8=-4$

Two complex solutions:
$\left\{\frac{1+i}{2},\frac{1-i}{2}\right\}$</td>
</tr>
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NOTE: the discriminate is the part UNDER the radical sign. It is just b^2-4ac.

TRY:

What is $b^2 – 4ac$? How many solutions and of what type will there be to these equations?

Equation	a	b	c	b^2-4ac ?	1 or 2 rational/irrational/complex?
$x^2+6x+9=0$	1	6	9	0	1 real solution
$x^2+10x+25=0$					
$-x^2+3x-4=0$	1	-3	4		Two complex solutions
$x^2+4x-12=0$					
$2x^2+3x+5=0$					
$3x^2+5x-1=0$					
$12-7x+x^2=0$	1	-7	12		

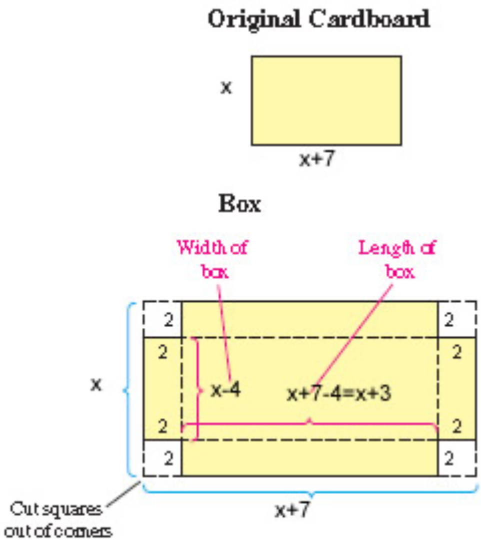
Quadratic Equation Applications

Solving for a Variable

Solve $V = \dfrac{\pi r^2 h}{3}$ for r. $3V = \dfrac{\pi r^2 h}{3} \cdot 3$ multiply by 3 to get: $3V = \pi r^2 h$ $\dfrac{3V}{\pi h} = \dfrac{\pi r^2 h}{\pi h}$ divide by πh to get: $\dfrac{3V}{\pi h} = r^2$ $\sqrt{\dfrac{3V}{\pi h}} = \sqrt{r^2}$ Square root property to get r $\pm\sqrt{\dfrac{3V}{\pi h}} = r$ Rationalize $\pm\dfrac{\sqrt{3V\pi h}}{\pi h} = r$ $\left\{-\dfrac{\sqrt{3V\pi h}}{\pi h}, \dfrac{\sqrt{3V\pi h}}{\pi h}\right\}$	TRY: Solve $K = \dfrac{xy^2}{z}$ for y.
Solve $V = \sqrt{\dfrac{3rt}{m}}$ for r. $V^2 = \left(\sqrt{\dfrac{3rt}{m}}\right)^2$ square both sides to get: $V^2 = \dfrac{3rt}{m}$ $V^2 m = \dfrac{3rt}{m} \cdot m$ multiply by m to get: $V^2 m = 3rt$ $\dfrac{V^2 m}{3t} = \dfrac{3rt}{3t}$ divide by $3t$ to get r $\dfrac{V^2 m}{3t} = r$ $\left\{\dfrac{V^2 m}{3t}\right\}$	TRY: Solve $Z = \sqrt{\dfrac{3xy}{w}}$ for x .

Solving an application problem:

BJ needs to create a box. He has a piece of cardboard that is 7 inches longer than it is wide. He will make the box by cutting out 2-inch squares from each corner and folding the sides up. Find the length and width of the original piece of cardboard if the volume of the finished box is 120 in^3.



X = original width

New width = Original width – corners cut out

W= x – 2 – 2 or W = x – 4

New length = 0riginal length – corners cut out

L = (x+7) – 2 – 2 or L=(x+7) – 4 or L=x+3

Height of the box = 2 .... The size of corner cut out

Volume = L · W · H

$120 = (x+3)(x-4)(2)$ Divide by 2

$60 = (x+3)(x-4)$ FOIL

$60 = x^2 - 4x + 3x - 12$ Combine

$0 = x^2 - x - 72$ Factor

$0 = x - 9$ or $0 = x + 8$

$9 = x$ or $-8 = x$ Only 9 makes sense.

The original width was 9 inches.

The original length was x+7 or 16 inches.

TRY:

BJ needs to create another box. This time the width of his piece of cardboard is 6 inches less than its length. If he cuts out 3-inch squares from each corner and turns the sides of the box up, the volume of the resulting box is 216 in^2. Find the length and width of the original piece of cardboard.

TRY:

Tom's garden is 20' by 30'. He wants to increase the length and width by the same amount to have a 1064 ft^2 garden. What should be the new dimensions of the garden?