

What if we had a 'magical' formula that was quicker and easier to use and would always work?

What if we could develop a formula that solve for x in the generic quadratic equation $ax^2 + bx + c = 0$?

Developing the Quadratic Formula

Instructor's note: Please listen to the lesson covering the following development of the quadratic formula. This is my favorite part of the entire course!

*Given the standard form of the Quadratic Equation – one can solve for x by completing the square. The resulting formula, known as the **Quadratic Formula**, will work to solve for any x .*

Given: $ax^2 + bx + c = 0$ where $a, b,$ and c are real numbers and $a \neq 0$

Use the five step involving completing the square and the square root property to solve for x .

1) Make the leading coefficient 1. Divide each term by a . $\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = 0$

2) Rewrite the equation: $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

3) Complete the Square: $x^2 + \frac{b}{a}x + \underline{\hspace{2cm}} = -\frac{c}{a} + \underline{\hspace{2cm}}$ Note: $\frac{1}{2}$ of $\frac{b}{a}$ is $\frac{b}{2a}$.

$$\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2} \quad \frac{b^2}{4a^2} \text{ was added to both sides.}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \quad \text{A common denominator } 4a^2 \text{ was found}$$

and the two terms on the right side combined.

4) Take the square root: $\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}} \rightarrow \left(x + \frac{b}{2a}\right) = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$

5) Isolate the x : $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$ and combine the right side terms: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

YEAH!!! This is a wonderful formula that solves for x in any quadratic equation! KEEP IT HANDY!