Solving Quadratics by Completing the Square

By blending the concept of Completing the Square and the Square Root property, quadratic equations that are not easily factored can now be easily solved.

Solving by Completing the Square $ax^2+bx+c=0$

1. If $a=1$, proceed to step 2. If $a\neq 1$, divide each term of the equation by a before completing the square.

2. Write the equation with the variable terms on one side and the constant on the other. That is, move c to the other side.
3. Determine $\frac{1}{2}$ of the b value. Square that and add it to each side of the equation.
4. Write the side with the terms containing the variable as a perfect square $\left(x-\frac{1}{2}b\right) or \left(x+\frac{1}{2}b\right)$ and combine terms on the other side.

5. Extract the roots using the square root property and solve the resulting linear equations.

Examples – Follow the 5 steps given above.

$x^2-6x-7=0$ $x^2-6x+\_\_=7+\_\_$ Move the -7 to the right side. $\frac{1}{2}(b)=\frac{1}{2}(6)=3$ and $3^2=9$ so add 9 to both sides. $x^2-6x+9=7+9$ $(x-3)^2=16$ $\sqrt{(x-3)^2}=\sqrt{16}$ Now use the square root property. $x-3=\pm 4$ $x=3+4$ or $x=3-4$ $x=7$ or $x=-1$ $\{-1,7\}$	$3x^2-6x-18=0$ Divide first by 3 to make $a=1$. $x^2-2x-6=0$ $x^2-2x+\_\_=6+\_\_$ Move the 6 to the right side. $\frac{1}{2}(b)=\frac{1}{2}(2)=1$ and $1^2=1$ so add 1 to both sides. $x^2-2x+1=6+1$ $(x-1)^2=7$ $\sqrt{(x-1)^2}=\sqrt{7}$ Use the square root property. $x-1=\pm\sqrt{7}$ $x=1\pm\sqrt{7}$ $\{1-\sqrt{7},1+\sqrt{7}\}$

TRY:

Solve by completing the square:

$y^2 - 3y - 10 = 0$ $\qquad\qquad$ $2m^2 - 8m - 20 = 0$

Sometimes …

…the solutions are not pretty.	…the equations are not pretty.	…the solutions are not real.
$4x^2 + 4x - 2 = 0$ Divide by 4. $x^2 + x - \frac{1}{2} = 0$ $x^2 + x + \_\_ = \frac{1}{2} + \_\_$ $x^2 + x + \frac{1}{4} = \frac{1}{2} + \frac{1}{4}$ $(x + \frac{1}{2})^2 = \frac{3}{4}$ $\sqrt{(x + \frac{1}{2})^2} = \sqrt{\frac{3}{4}}$ $x + \frac{1}{2} = \pm\sqrt{\frac{3}{4}}$ $x + \frac{1}{2} = \pm\frac{\sqrt{3}}{2}$ $x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}$ $x = \frac{-1 \pm \sqrt{3}}{2}$ $\left\{\frac{-1-\sqrt{3}}{2}, \frac{-1+\sqrt{3}}{2}\right\}$	Put in proper form first. $2s(2s + 5) - 20 = 18s$ $4s^2 + 10s - 20 - 18s = 0$ $4s^2 - 8s - 20 = 0$ Divide by 4. $s^2 - 2s - 5 = 0$ $s^2 - 2s + \_\_ = 5 + \_\_$ $s^2 - 2s + 1 = 5 + 1$ $(s - 1)^2 = 6$ $\sqrt{(s-1)^2} = \sqrt{6}$ $s - 1 = \pm\sqrt{6}$ $s = 1 \pm \sqrt{6}$ $\{1 - \sqrt{6}, 1 + \sqrt{6}\}$	$x^2 - 4x + 12 = 0$ $x^2 - 4x + \_\_ = -12 + \_\_$ $x^2 - 4x + 4 = -12 + 4$ $(x - 2)^2 = -8$ $\sqrt{(x-2)^2} = \sqrt{-8}$ $x - 2 = \pm\sqrt{-8}$ $x = 2 \pm \sqrt{-8}$ $x = 2 \pm i\sqrt{8}$ $\quad \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$ $x = 2 \pm 2i\sqrt{2}$ $\{2 - 2i\sqrt{2}, 2 + 2i\sqrt{2}\}$

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