

Exponent: 1/n

Rational Exponents

Consider the following:

$$\begin{aligned}\sqrt{4} &= \sqrt{2^2} = 2^1 = 2 & \sqrt[3]{8} &= \sqrt[3]{2^3} = 2^1 = 2 \\ \sqrt{16} &= \sqrt{2^4} = 2^2 = 4 \\ \sqrt{64} &= \sqrt{2^6} \text{ or } \sqrt{64} = 8 \text{ or } \sqrt{2^6} = 2^3 \\ \sqrt[3]{64} &= \sqrt[3]{2^6} \text{ or } \sqrt[3]{64} = 4 \text{ or } \sqrt[3]{2^6} = 2^2\end{aligned}$$

What is the relationship between the power (the exponent) and the index (the root) and the exponent of the answer?

What is $\sqrt{4^6} = \underline{\hspace{2cm}}$ What is $\sqrt[3]{10^{12}} = \underline{\hspace{2cm}}$

What have you discovered? $\sqrt[n]{x^m} = x^{m/n} = x^{\frac{\text{power}}{\text{root}}}$

The numerator (m) indicates the **power** to which the base is to be raised, and
The denominator (n) indicates the **index [root to be taken]**.

If the rational exponent can be reduced, do so. $\sqrt[4]{3^8} = 3^{8/4} = 3^2 = 9$

Definition of $a^{1/n}$

If n is any positive integer, then $a^{1/n} = \sqrt[n]{a}$, provided that $\sqrt[n]{a}$ is a real number.

$$256^{1/4} = \sqrt[4]{256} = 4 \qquad (-8)^{1/3} = \sqrt[3]{-8} = -2 \qquad \left(\frac{81}{16}\right)^{1/4} = \sqrt[4]{\frac{81}{16}} = \frac{\sqrt[4]{81}}{\sqrt[4]{16}} = \frac{3}{2}$$

TRY:

Rewrite as a radical and evaluate.

$$16^{1/2}$$

$$(-32)^{1/5}$$

$$\left(\frac{27}{125}\right)^{1/3}$$