Lesson 18: Roots, Radicals, Rational Exponents

Definitions: Roots, Radicals

Definition of *n*th Root

If $a = b^n$ for a positive integer *n*, then *b* is the *n*th root of *a*. $\sqrt[n]{a} = b$

If $a = b^2$, then *b* is a square root of *a*. $\sqrt{a} = b$

If $a = b^3$, then *b* is the cube root of *a*. $\sqrt[3]{a} = b$

3 is a second (square) root of 9 since $3^2 = 9$ and $\sqrt{9} = 3$

-2 is the third (cube) root of –8 since $(-2)^3 = -8$ and $\sqrt[3]{-8} = -2$

Radicals

radical symbol $\sqrt[n]{a}$ mathematical sign used to signify roots that is: $\sqrt[index]{radicand}$

a is called the **radicand**, *n* is the **index** (or root) of the radical.

The entire expression $\sqrt[n]{a}$ is called a **radical**.

If there is no index associated with the radical symbol, it is understood to be **2**. Ex: $\sqrt{25}$

Note: If the radicand (the number under the radical sign) is *negative* and the index is *even*, the radical does not represent a real number. Ex: $\sqrt{-4}$ is not a real number. Why?

To find the square root of a number, ask what number squared equals that number.

What squared equals 49? What multiplied by itself equals 49? $7 \cdot 7 = (7)^2 = 49$ so $\sqrt{49} = 7$

The $\sqrt{\ }$ symbol represents ONLY the positive square root $\sqrt{49} = 7$ even though $(-7)^2 = 49$.

TRY: $\sqrt{64}$ $-\sqrt{25}$ $\sqrt{-9}$

What number cubed equals 27?

What multiplied by itself three times equals 27?

$3 \cdot 3 \cdot 3 = (3)^3 = 27$ or $\sqrt[3]{27} = 3$

$\sqrt[3]{27}$ is stated "The third root of 27" or "the cube root of 27".

TRY: $\sqrt[3]{8}$ $\sqrt[3]{-1}$ $\sqrt[3]{-125}$

Sometimes, the radicand is a rational expression. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $\sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$

TRY: $\sqrt{\frac{36}{100}}$ $-\sqrt{\frac{49}{25}}$