Rational Equations: Work Problems

Work Rate:

If a job can be completed in t units of time, then the rate of work is 1/t of the job per unit of time.

Work Equation:

If T represents the time to complete the job, and t_1 represents the amount of time necessary for unit 1 (person1) to complete the job, and t_2 represents the amount of time necessary for unit 2 (person2) to complete the job, and so on, then to find out how long it would take all the units (all the people) working together to complete the job, solve the following equation for T:

$1/t_1 + 1/t_2 + \ldots = 1/T$ or $(1/t_1)T + (1/t_2)T + \ldots = (1/T)T$

Multiplying each part by T, results in 1 job completed.

1. Mickey can clean the game room in 20 minutes. Terry can clean the game room in 30 minutes. Find how long it will take them to clean the game room if they work together.

Method 1 set up – what does it take to accomplish 1complete job?

$\frac{1}{20}T + \frac{1}{30}T = 1$ LCD is 60 $\frac{1(60)}{20}T + \frac{1(60)}{30}T = 1(60)$ $3T + 2T = 60$ $5T = 60$

$T = 12$ minutes

Method 2 set up – what portion of the total job will be completed in one unit of time? For example, if it takes 20 minutes to do the job, then 1/20 of the job is completed in 1 minute and 1/T of the job is completed in 1 minute. [Personally, I like to use this method. You may use either.]

$\frac{1}{20} + \frac{1}{30} = \frac{1}{T}$ LCD is 60T $\frac{1(60T)}{20} + \frac{1(60T)}{30} = \frac{1(60T)}{T}$ $3T + 2T = 60$ $5T = 60$

$T = 12$ minutes

2. The old printing press took twice as long as the new press to print newspapers. Together it takes 12 hours to print the papers. How long does it take the old press to do the job alone?

T = new press time 2T = old press time

$\frac{1}{T} + \frac{1}{2T} = \frac{1}{12}$ (using Method 2)

3. Using the green hose one can fill a small pond in 45 minutes. If both the blue and the green hose are used, one can fill the same pond in 20 minutes. How long would it take to fill the pond using only the blue hose?

T = Blue hose time $\frac{1}{45} + \frac{1}{T} = \frac{1}{20}$ (using Method 2)

TRY:

1. Three postal workers can sort a stack of mail in 20 minutes, 30 minutes, and 60 minutes, respectively. How long will it take them to sort the mail if all three work together?

2. It takes Alan twice as long to prepare the lab beakers as Steve. With Steve's help, the beakers can be processed in 4 hours. How long does it take Steve to prepare the beakers alone?

3. Julie can paint a fence by herself in 12 hours. With Betsy's help, it takes only 5 hours. How long would it take Betsy by herself?