Math 20   Unit #3   5.4; 2.2 to 2.4; 6.1 to 6.4; 7.1 to 7.3
To the Test—be sure to bring:
   (1) your personally-prepared 8 ½" by 11" study guide for this test
   (2) your simple, non-graphing calculator and
   (3) your pencils
   (4) your BluGold ID

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1. Develop the equations you need to solve this problem, list them, and solve the problem.
    One serving of Coke and two servings of Mountain Dew contain 31.3 tsp of sugar while one serving of Mountain Dew and three servings of Coke contain 38.9 tsp of sugar. How much sugar is in one serving of each drink?

$$1C + 2D = 31.3$$
$$3C + 1D = 38.9$$

$$-3C - 6D = -93.9$$
$$3C + 1D = 38.9$$
$$\overline{}$$
$$\frac{-5D}{-5} = \frac{-55}{-5}$$
$$D = 11$$

$$1C + 2(11) = 31.3$$
$$1C + 22 = 31.3$$
$$\underline{\quad -22 = -22}$$
$$1C = 9.3$$

Coke has 9.3 tsp of sugar and Mountain Dew has 11 tsp. of sugar.

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2. How many milliliters of a 4% acid solution and how many milliliters of a 10% acid solution must be mixed together to obtain 54 milliliters of a 6% solution?

*if the amount of 10% were known, it would go here and the total would be that amount + x

	mL	%	part	Equation
	X	.04	.04X	
*	(54-x)	.10	.10(54-x)	
Total ⇒	54	.06	.06(54)	

Equation
$$.04x + .10(54-x) = .06(54)$$ multiply each term by 100
$$4x + 10(54-x) = 6(54)$$ ↑one term
$$4x + 540 - 10x = 324$$
$$-6x + 540 = 324$$
$$-6x = -216$$

$$\frac{6x}{6} = \frac{216}{6}$$
$$x = 36$$

$$\begin{array}{r} 54 \\ -36 \\ \hline 18 \end{array}$$

(36 ml of 4% and 18 ml of 10%)

CJ invested $4000 in two accounts, some of it at 5% simple interest and the rest of it at 3% simple interest. How much did she invest in each account if she earned $144 in interest after one year?

$	%	int
x	.05	$.05x$
$(4000-x)$	.03	$.03(4000-x)$
4000		144

$.05x + .03(4000-x) = 144$

$5x + 3(4000-x) = 14400$

$5x + 12000 - 3x = 14400$

$2x + 12000 = 14400$

$\quad\quad -12000 = -12000$

$\overline{\quad 2x \quad\quad\quad = 2400\quad}$

$\dfrac{2x}{2} = \dfrac{2400}{2}$

$x = 1200$

$\begin{array}{r} 4000 \\ -1200 \\ \hline 2800 \end{array}$

put $1200 at 5%
and $2800 at 3%

3. A small plane can fly 400 miles in the same amount of time a jet can fly 1000 miles. If the jet's speed is 300 mph faster than the small plane, find the speed of the jet and the speed of the small plane.
   [d = r * t]

$\dfrac{d}{r} = \dfrac{r \cdot t}{r}$

$\dfrac{d}{r} = t$

so

$\dfrac{d \text{ of small plane}}{r \text{ of small plane}} \overset{\text{same time}}{=} \dfrac{d \text{ of jet}}{r \text{ of jet}}$

$\dfrac{400}{x} = \dfrac{1000}{x+300}$

$400(x+300) = 1000x$

$400x + 120000 = 1000x$

$-400x \quad\quad = -400x$

$\dfrac{120000}{600} = \dfrac{600x}{600}$

$200 = x$

Small plane goes 200 mph. Jet goes 500 mph.

4. Evaluate:

-5^2 $-(5^2)$ $(-5)^2$ -5^0 $(-5)^0$

$= 5 \cdot 5$ $= (5 \cdot 5)$ $(-5)(-5)$

-25 -25 25 -1 1

5. Simplify. Assume all variables represent nonzero real numbers.

$$6ab^2(9a^7b^{10})$$

$$\boxed{54a^8b^{12}}$$

multiply coefficients
Add exponents
on like bases

$$\frac{(-5d^6)^3}{(5c^3)^2}$$

power raised to power ⟹ multiply exponents

$$\frac{(-5)^3 d^{18}}{5^2 c^6}$$

$$\frac{-125d^{18}}{25\,c^6} = \boxed{\frac{-5d^{18}}{c^6}}$$

$$\left(\frac{3}{8}x^3y^4\right)^3\left(\frac{8}{9}x^5y^3\right)^2$$

$$\frac{3^3}{8^3}x^9y^{12}\cdot\frac{8^2}{9^2}x^{10}y^6$$

$$\frac{27}{8}\cdot\frac{1}{\cancel{8}^{\,3}}\,x^{19}y^{18}$$

$$\boxed{\frac{x^{19}y^{18}}{24}}$$

you may have different steps but the answer should match.

6. Simplify.

$$\left(\frac{2}{3}\right)^{-5}$$

$$\frac{2^{-5}}{3^{-5}} = \frac{3^5}{2^5}$$

$$= \frac{3\cdot3\cdot3\cdot3\cdot3}{2\cdot2\cdot2\cdot2\cdot2}$$

$$= \boxed{\frac{243}{32}}$$

$$\left(\frac{12}{5}\right)^{-2}$$

$$\frac{12^{-2}}{5^{-2}} = \frac{5^2}{12^2}$$

$$= \boxed{\frac{25}{144}}$$

$$4^{-2} - 8^{-1}$$

$$\frac{1}{4^2} - \frac{1}{8^1}$$

$$= \frac{1}{16} - \frac{1}{8}$$

$$= \frac{1}{16} - \frac{2}{16}$$

$$= \boxed{-\frac{1}{16}}$$

7. Simplify. Assume all variables represent nonzero real numbers.
    The answer should contain only positive exponents.

$$7\left(\frac{1}{x}\right)^{-5}$$

$$\frac{63a^2b^{-2}}{9c^{-7}d^{-10}}$$

$$7 \cdot \left(\frac{1^{-5}}{x^{-5}}\right)$$

$$7 \cdot \frac{x^5}{1^5}$$

$$\boxed{7x^5}$$

$$\frac{7 \cancel{63} a^2 b^{-2}\downarrow}{\cancel{9} c^{-7} d^{-10}}$$

$$\boxed{\frac{7a^2c^7d^{10}}{b^2}}$$

8. Simplify. Assume all variables represent nonzero real numbers.
    The answer should not contain negative exponents.

$$\frac{63a^2b^{-2}c^5}{9a^{-7}b^{-10}c^3}$$

$$\frac{63 a^2 b^{-2} c^5}{9 a^{-7} b^{-10} c^3}$$

$$\frac{7\cancel{63} a^{2--7} b^{-2--10} c^{5-3}}{\cancel{9}}$$

OR

$$\frac{7\cancel{63} a^2 a^7 b^{\overset{8}{10}} c^{\overset{2}{\cancel{8}}}}{\cancel{9} \quad b^{\cancel{2}} \quad c^{\cancel{3}}}$$

$$= \boxed{7 a^9 b^8 c^2}$$

9. Simplify. Assume all variables represent nonzero real numbers.
   The answer should not contain negative exponents.

Text suggests
$\left(\dfrac{5mn}{3m^8n^{-4}}\right)$ $\overset{flip}{\underset{and}{2.}}$

$$\left(\frac{3m^8n^{-4}}{5mn}\right)^{-2}$$

$$\frac{3^{-2}\,m^{-16}\,n^{8}}{5^{-2}\,m^{-2}\,n^{-2}}$$

$$\frac{5^{2}\,m^{2}\,n^{8}\,n^{2}}{3^{2}\,m^{16}}$$

$$\boxed{\frac{25\;\;n^{10}}{9\;\;m^{14}}}$$

$$\left(\frac{5a^2b^{-2}c^5}{15a^{-7}b^{-10}c^3}\right)^{-3}$$

$$\frac{5^{-3}\,a^{-6}\,b^{6}\,c^{-15}}{15^{-3}\,a^{21}\,b^{30}\,c^{-9}}$$

$$\frac{15^{3}\,b^{6}\quad c^{9}}{5^{3}\,a^{21}a^{6}\,b^{30}\,c^{15}}$$

$$\boxed{\frac{27}{a^{27}\,b^{24}\,c^{6}}}$$

Think
$\dfrac{\cancel{15}\cdot\cancel{15}\cdot\cancel{15}}{\cancel{5}\cdot\cancel{5}\cdot\cancel{5}}$ 3 3 3

10. Perform the indicated operations:

$$4x - (-6x + 8)$$

$$4x + 6x - 8$$

$$\boxed{10x - 8}$$

$$(5x^2 - 6x - 8) + (3x^2 - 9x + 15)$$

$$+\ 3x^2 - 9x + 15$$

$$\boxed{8x^2 - 15x + 7}$$

$$(-x^3 - 4x^2 + 3x - 9) - (5x^2 - 21) - (-4x^3 + 3x + 9)$$

$$-x^3 - 4x^2 + 3x - 9$$
$$-5x^2 \qquad\qquad +21$$
$$+4x^3 \qquad\quad -3x - 9$$

$$\boxed{3x^3 - 9x^2 + 3}$$

11. Find the product.

$(5x+4)(4x-5)$

$$20x^2 - 25x + 16x - 20$$
$$\boxed{20x^2 - 9x - 20}$$

$(5x+4)^2$

$$(5x+4)(5x+4)$$
$$\boxed{25x^2 + 40x + 16}$$

$(3x-8)(7x+1)$

$$21x^2 + 3x - 56x - 8$$
$$\boxed{21x^2 - 53x - 8}$$

$(5x-4)^2$

$$(5x-4)(5x-4)$$
$$\boxed{25x^2 - 40x + 16}$$

12. Find the product.

$-3x(4x-5)$

$$\boxed{-12x^2 + 15x}$$

$-3x(\frac{2}{3}x+5)^2$ ← careful

$$-3x\left(\frac{2}{3}x+5\right)\left(\frac{2}{3}x+5\right)$$

$$(-2x^2 - 15x)\left(\frac{2}{3}x+5\right)$$

$$-\frac{4x^3}{3} - 10x^2 - \frac{30x^2}{3}^{10} - 75x$$

$$\boxed{-\frac{4}{3}x^3 - 20x^2 - 75x}$$

$(4x^2y^3)(-6x^2y^5)$

$$\boxed{-24x^4y^8}$$

13. Divide.

$$\frac{-12a^3 + 9a^2 - 21a}{-3a}$$

$$= \frac{-12a^3}{-3a} + \frac{9a^2}{-3a} - \frac{21a}{-3a}$$

$$= \boxed{4a^2 - 3a + 7}$$

14. What is the Greatest Common Factor (GCF) of $15x^2$ and $10x$?

$$3 \cdot 5 \cdot x \cdot x \qquad 2 \cdot 5 \cdot x$$

$$5x$$

Find the **G**reatest **C**ommon **F**actor.

$$5 \cdot 2 \cdot 3 \cdot b \cdot b \cdot b$$
$$30b^3 - 5b \qquad 5 \cdot b$$

$$5b$$

$$5x(2x - 3) + 8(2x - 3)$$

$$(2x - 3)$$

15. Factor by grouping.

$$dx - dy + 5x - 5y$$

$$d(x-y) + 5(x-y)$$

$$(x-y)(d+5)$$

16. Factor by grouping.

$$10y^2 - 8yz - 15yz + 12z^2$$

$$2y(5y - 4z) \quad - 3z(5y - 4z)$$

$$(5y - 4z)(2y - 3z)$$

$$ax^2 + bx + c$$

17. Factor completely by grouping.

$$8n^2 - 14n + 3$$

$m \cdot n = a \cdot c = 8 \cdot 3 = 24$

$m + n = b = -14$

$m, n = -12, -2$

$$8n^2 - 12n - 2n + 3$$

$$4n(2n-3) - 1(2n-3)$$

$$(2n-3)(4n-1)$$

$$6v^2 - 19v + 14$$

$m \cdot n = a \cdot c = 6 \cdot 14 = 84$

$m + n = b = -19$

		sum
6	14	20
2	41	43
3	28	31
7	12	19!

$m, n = -7, -12$

$$6v^2 - 7v - 12v + 14$$

$$v(6v-7) - 2(6v-7)$$

$$(6v-7)(v-2)$$

18. Factor completely.

$$x^2 - 11x + 24$$

$m \cdot n = 24$

$m + n = -11$

$$(x-3)(x-8)$$
$-3, -8$

$$x^2 + 11x + 24$$

$m \cdot n = 24$

$m + n = 11$

$$(x+3)(x+8)$$
$+3, +8$

19. Factor completely.

$x^2 - 2x - 24$

$m \cdot n = -24$
$m + n = -2$

$-6, 4$

$(x-6)(x+4)$

$x^2 + 2x - 24$

$m \cdot n = -24$
$m + n = 2$

$+6, -4$

$(x+6)(x-4)$

$x^2 - 10x - 24$

$m \cdot n = -24$
$m + n = -10$

$-12, +2$

$(x-12)(x+2)$