# Caution with Rational Expressions

## Thoughts to Think when approaching Rational Expressions including Cautions about Common Errors

Looking at a single fraction

- Factor the numerator if possible; factor the denominator if possible; reduce – cancel only the factors
  $\frac{2-4x}{3-4x} \neq \frac{2}{3}$ one CANNOT cancel the $-4x$ because $-4x$ is not a factor!
    - $\frac{2-4x}{6-8x} = \frac{2(1-2x)}{2(3-4x)} = \frac{1-2x}{3-4x}$ one CAN cancel the 2's. In the final answer, one CANNOT reduce $-2x$ and $-4x$ because they are not factors!
    - $\frac{21x^3}{35ax^2} = \frac{3 \cdot 7 \cdot x \cdot x \cdot x}{5 \cdot 7 \cdot a \cdot x \cdot x} = \frac{3x}{5a}$ single terms are composed of factors
    - $\frac{x^2-x-12}{x^2+5x+6} = \frac{(x+3)(x-4)}{(x+3)(x+2)} = \frac{x-4}{x+2}$ One CANNOT cancel out the xs or 2s
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Looking at two rational terms (meaning two fractions)

- If you are to MULTIPLY or DIVIDE two fractions, process each rational term (fraction) following the steps above. Then…
    - To multiply, after reducing each individual fraction, look to see if anything in either numerator cancels with anything in either denominator. If so, reduce. Multiply the remaining factors together for the final answer.
      $$\frac{2x^2+5x-12}{x^2+2x} \cdot \frac{x^3-3x^2}{2x^2-9x+9} = \frac{(2x-3)(x+4)}{x(x+2)} \cdot \frac{x^2(x-3)}{(2x-3)(x-3)} = \frac{x(x+4)}{(x+2)}$$
    - To divide, multiply by the reciprocal of the second fraction (turn it upside down), then reduce each individual fraction as much as possible. After reducing each individual fraction, look to see if anything in either numerator cancels with anything in either denominator. If so, reduce. Multiply the remaining factors together for the final answer.
      $$\frac{2x^2+5x-12}{x^2+2x} \div \frac{x+3}{x-3} = \frac{(x+2)(x+3)}{x(x+2)} \cdot \frac{(x-3)}{(x+3)} = \frac{(x+3)}{x} \cdot \frac{(x-3)}{(x+3)} = \frac{x-3}{x}$$

- To ADD or SUBTRACT two fractions:

    - IF the two denominators are already the same, just combine the numerators, then reduce.

      $\frac{9-4y}{3y}+\frac{6-y}{3y}=\frac{15-5y}{3y}$ One cannot reduce by 3 or by y as these are NOT factors of both the numerator and the denominator. Be careful when SUBTRACTING two fractions. Distribute the subtraction sign (a negative) to all terms in the numerator of the second fraction. Notice the 3 cannot be canceled until it is factored out first.

      $$\frac{9+4y}{3y}-\frac{3+y}{3y}=\frac{9+4y-3-y}{3y}=\frac{6+3y}{3y}=\frac{3(2+y)}{3y}=\frac{2+y}{y}$$

    - IF the two denominators are NOT the same, reduce each fraction as much as possible. Determine the LCD that must be used. Multiply each numerator by the factors in the LCD that are NOT in that fraction's denominator. Combine all numerators OVER the LCD – the denominator. Factor the combined numerator to see if you can reduce further.

      $\frac{2}{x^2-9}+\frac{3}{x^2-6x+9}=\frac{2}{(x+3)(x-3)}+\frac{3}{(x-3)(x-3)}=$ LCD is $(x+3)(x-3)^2$

      $$\frac{2(x-3)+3(x+3)}{(x+3)(x-3)^2}=\frac{2x-6+3x+9}{(x+3)(x-3)^2}=\frac{5x+3}{(x+3)(x-3)^2}$$

When two factors are opposite in signs:

- Factor out a -1 in one of the factors. $\frac{a-5}{5-a}=\frac{-1(5-a)}{5-a}=-1$ BE VERY CAREFUL to check to see if you have factored correctly. Sometimes, if the two terms within the expression are switched around, it may appear necessary to remove a -1, but it isn't. After factoring, reduce is possible.

  $$\frac{-7y+7x}{x}\bullet\frac{1}{x-y}=\frac{7x-7y}{x}\bullet\frac{1}{x-y}=\frac{7(x-y)}{x(x-y)}=\frac{7}{x}$$

  (No need to factor out a -1.)

 