Rational Expressions: Lowest Terms

Basic Principle of Rational Numbers

If $\dfrac{a}{b}$ is a rational number and c is a nonzero real number, then $\dfrac{a}{b} = \dfrac{a}{b} \cdot \dfrac{c}{c}$ and $\dfrac{a}{b} \cdot \dfrac{c}{c} = \dfrac{a}{b}$

That is, both the numerator and the denominator of a rational expression may be multiplied or divided by the same nonzero polynomial without changing the value of the expression.

To Write a Rational Expression in Lowest Terms

1. Completely factor the numerator and the denominator.

2. Divide the numerator and the denominator by the greatest common factor. (One may need to factor out a common factor with a negative sign to get identical factors in the numerator and the denominator.)

$$\frac{x^2 + 5x}{xy + 5y} = \frac{x(x+5)}{y(x+5)} = \frac{x}{y} \cdot \frac{(x+5)}{(x+5)} = \frac{x}{y} \cdot 1 = \frac{x}{y}$$

CAUTION: $\dfrac{x+5}{y+5} \neq \dfrac{x}{y}$ One can ONLY reduce factors!

TRY:

Simplify each of the following rational expressions.

$$\frac{3a+3}{3} \qquad\qquad \frac{7x-14}{7x} \qquad\qquad \frac{b^8 - ab^5}{ab^5} \qquad\qquad \frac{2a^2 - 2b^2}{2a^2 + 2b^2}$$