Lesson 16: Rational Expressions

Rational Expressions: Domain

Rational Expression

A **rational expression** is any algebraic expression that can be written as a quotient of two polynomials where the denominator is not equal to zero.

$\frac{3}{7}$ (ratio of 2 monomials) $z+6$ (understood denominator of 1)

Consider: $\frac{x+3}{y-7}$ When is this fraction undefined?

$y \neq 7$ because 7 will cause the denominator to be 0.

Therefore, a restriction is placed on the variable in the denominator. One can use any number for y except 7.

> **Restrictions** on any variable in the denominator are found by setting all factors of the denominator, that contain the variable, equal to zero and solving the resulting equation(s) for the variable. Values of the variable that make the denominator zero are ***excluded*** from the domain.
>
> Therefore, the **domain** of a rational expression is the set of all replacement values of the variable for which the expression is **defined**. Stated another way, the domain of a rational expression is the set of all real numbers except the value(s) of the variable that make(s) the denominator equal 0.

In $\frac{x+3}{y-7}$, if $y=7$ then the denominator would be 0.

The domain may be stated two ways:

$\{y \mid y \neq 7\}$ in set-builder notation or $(-\infty,7)\cup(7,\infty)$ in interval notation

Finding the Domain of a Rational Expression

1. Completely factor the denominator.

2. Set each factor of the denominator containing the variable equal to zero (using the zero product property where "given real numbers p and q, if p·q = 0, then p = 0 or q = 0).

3. Solve the resulting equations. The solutions are the restrictions placed on the variable. The solutions are *excluded* from the domain.

State the **domain** in set-builder and interval notation.

$$\frac{3b+1}{b^2-3b}$$

Factors of the denominator $(b)(b-3)$

$\{b \mid b \neq 0, b \neq 3\}$ in set-builder notation

$(-\infty,0)\cup(0,3)\cup(3,\infty)$ in interval notation

$$\frac{y+5}{y^2+9}$$

Denominator will not factor.

y^2 will always be positive

$\{y \mid y \in \mathrm{Reals}\}$ $(-\infty,\infty)$

TRY:

$$\frac{2y-1}{y^2-9}$$