Quadratic Equations, Zero Product Property

Quadratic Equation

The **standard form** of a **quadratic equation** is $ax^2 + bx + c = 0$
where a, b, and c are real numbers and $a \neq 0$.

Question: If you were told 5 times some number equals 0, what would the other number be?
If 5x = 0 what must be true about x?
If wxy = 0, what must be true about w or x or y?
If $(x+3)(x-5) = 0$, what must be true about $(x+3)$ or $(x-5)$?

Zero Product Property

Given real numbers p and q, if pq = 0, then p = 0 or q = 0.

Solving a Quadratic Equation by Factoring

1. Write the quadratic equation in standard form ($ax^2 + bx + c = 0$) with the leading coefficient *positive*.

   If the first term is negative, multiply every term of the equation by -1 to make it positive.

2. Completely factor the quadratic expression.

3. Use the zero factor property to set each of the factors containing the variable equal to 0.

4. Solve the simpler linear equations.

5. Check the solution(s) in the original equation.

Solve for x: $x^2 - 2x - 15 = 0$
Factor: $(x+3)(x-5) = 0$
Set each factor to 0 $(x+3) = 0$ or $(x-5) = 0$
Solve each $x = -3$ or $x = 5$
Solution: $\{-3, 5\}$ written from smallest to largest

Be sure to check the solutions in the original equation.

$x^2-2x-15=0$ $\quad$ $x^2-2x-15=0$

$(-3)^2-2(-3)-15=0$ $\quad$ $(5)^2-2(5)-15=0$

$9+6-15=0$ $\quad$ $25-10-15=0$

$0=0$ $\quad$ $0=0$

Solve these already factored equations:

$(3p+8)(4p-3)=0$

$(3p+8)=0$ or $(4p-3)=0$

$3p=-8$ or $4p=3$

$p=-\frac{8}{3}$ or $p=\frac{3}{4}$

$\left\{-\frac{8}{3},\frac{3}{4}\right\}$

$8(x-9)(x+9)=0$ Divide both sides by 8 first.

$(x-9)(x+9)=0$

$(x-9)=0$ or $(x+9)=0$

$x=9$ or $x=-9$

$\{-9,9\}$

Consider:

$5x^2+40x+60=0$ One **can** DIVIDE both sides by 5 to simplify the problem to: $x^2+8x+12=0$

TRY to solve: $x^2+8x+12=0$

$6x^2=-36x$ One **can** DIVIDE both sides by 6 to simplify the problem to: $x^2=-6x$

One **CANNOT** divide both sides by 'x' as that would eliminate an unknown solution, 0 in this case.

In general, one can divide an equation by a nonzero real number, but not by a variable.

Solve:	$6x^2=-36x$
Rewrite in standard form:	$6x^2+36x=0$
Divide both sides by the nonzero number, 6:	$x^2+6x=0$ 0 divided by 6 is still 0
Factor out the GCF:	$x(x+6)=0$
Set each of the factors to 0	$x=0$ or $(x+6)=0$
Solve	$x=0$ or $x=-6$
Solution:	$\{-6,0\}$ Check the solutions.

TRY these: Don't forget to remove the GCF first!
Be sure the equation is in standard form … 1st term positive and all set to 0.

$t^2 - 6t - 27 = 0$

$h^2 - 5h = 0$ hint: factor out the GCF

$p^2 - p = 42$ hint: rewrite to standard form

$-2x^2 - 16x - 24 = 0$ hint: 1st term must be positive; multiply all by -1

$x^3 - 16x = 0$ hint: factor completely

$w^3 - w^2 - 25w + 25 = 0$ hint: factor by grouping