

Sum and Difference of Two Cubes

Factoring the Difference of Two Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

1. Identify that we have a perfect cube minus another perfect cube.
2. Rewrite the problem as a first term cubed minus a second term cubed.
 $(1^{\text{st}} \text{ term})^3 - (2^{\text{nd}} \text{ term})^3$
3. Factor the problem into the (first term minus the second term) times [the first term squared plus (the first term times the second term) plus the second term squared].

$$(1^{\text{st}} \text{ term} - 2^{\text{nd}} \text{ term}) [(1^{\text{st}} \text{ term})^2 + (1^{\text{st}} \text{ term} \cdot 2^{\text{nd}} \text{ term}) + (2^{\text{nd}} \text{ term})^2]$$

$$y^3 - 125 = (y)^3 - (5)^3$$

Substitute y for the "a".

Substitute 5 for the "b".

$$(a - b)(a^2 + ab + b^2)$$

$$(y - 5)(y^2 + 5y + 5^2) = (y - 5)(y^2 + 5y + 25)$$

Factoring the Sum of Two Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

1. Identify that we have a perfect cube plus another perfect cube.
2. Rewrite the problem as a first term cubed plus a second term cubed.
 $(1^{\text{st}} \text{ term})^3 + (2^{\text{nd}} \text{ term})^3$
3. Factor the problem into the (first term plus the second term) times [the first term squared minus (the first term times the second term) plus the second term squared].

$$(1^{\text{st}} \text{ term} + 2^{\text{nd}} \text{ term}) [(1^{\text{st}} \text{ term})^2 - (1^{\text{st}} \text{ term} \cdot 2^{\text{nd}} \text{ term}) + (2^{\text{nd}} \text{ term})^2]$$

$$y^3 + 125 = (y)^3 + (5)^3$$

Substitute y for the "a".

Substitute 5 for the "b".

$$(a + b)(a^2 - ab + b^2)$$

$$(y + 5)(y^2 - 5y + 5^2) = (y + 5)(y^2 - 5y + 25)$$

Hint:

In the factorization of both the sum and the differences of two cubes, the sign in the binomial factor is **the same** as the sign of the second term of the binomial factor.

The sign of the middle term of the trinomial factor has the **opposite** sign of the second term of the binomial factor.

$$\text{Difference: } a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$\text{Sum: } a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Cubed Forms

$x^{15} - 27$ Can be thought of as: $(x^5)^3 - (3)^3$ then apply the rule.

$$(x^5 - 3)(x^{10} + 3x^5 + 9)$$

TRY:

How can one think of $8x^6 - 1000y^{12}$

Careful – finding the cube of a coefficient is different than finding the cubed of a variable with exponent.