Lesson 13: Exponent Rules

Definitions: Exponent, Base

Think about the following: a^3 What does it mean?

In the **exponential** expression a^3 (read "a to the third power"), **a** is called the **base** and **3** is called the **exponent.**

Definition: If *a* is a nonzero real number and *n* is a positive integer, then $a^n = a \cdot a \cdot a \cdot \ldots \cdot a$

$$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$$

TRY: 4^3

Be careful: Expressions of the form $(-2)^4$ and -2^4 These expressions are not always equal.

$(-2)^4 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) = 16$ $\qquad$ $-2^4 = -2 \cdot 2 \cdot 2 \cdot 2 = -16$

The placement of the ( ) makes a difference.

TRY: $(-3)^2$ and -3^2

Product Rule for Exponents

Product Rule for Exponents

If *m* and *n* are integers and *a* ≠ 0, then, $a^m \cdot a^n = a^{m+n}$

i.e. When multiplying expressions with **like bases**, add the exponents to get the exponent of the common base. If the base is a number, remember NOT to multiply the base.

$3^4 \cdot 3^2 = 3^{4+2} = 3^6 = 729$ $\qquad$ $-2^2 y^2 (5y^4) = -20y^6$

$x^4 \cdot y^2 = x^4 y^2$ Since the bases are different, the factors cannot be combined.

TRY: $4x^3(5x)$

Power Rules

Power of a Product Rule

If *a* and *b* are nonzero real numbers and *n* is any integer, then $(ab)^n = a^n b^n$

i.e. When a group of factors is raised to a power, raise each of the **factors** in the group to this power.

WARNING: $(xy)^5 = x^5y^5$ and $(2ab)^4 = 2^4a^4b^4$ but $(2+3)^3 \neq 2^3 + 3^3$ because 2 and 3 are terms, NOT factors!

$(2+3)^3 = (2+3)(2+3)(2+3)$ or in general $(a+b)^3=(a+b)(a+b)(a+b)$

$(-2x)^2 = (-2)^2(x)^2 = 4x^2$ This is different than $-2^2x^2 = -4x^2$

TRY: $(3y)^3$ $(-3y)^3$ $-(3y)^3$

Power of a Power Rule

If *m* and *n* are any integers and $a \neq 0$, then $(a^m)^n = a^{m \bullet n}$

i.e. A power to a power is found by multiplying the exponents.

$(2^3)^2 = 2^{3 \cdot 2} = 2^6 = 64$ $(2x^2)^4 = 2^{1 \cdot 4}x^{2 \cdot 4} = 2^4x^8 = 16x^8$ $\dfrac{(3a^2)^3}{(6a^3)^2} = \dfrac{3^3a^6}{6^2a^6} = \dfrac{3^3}{6^2} = \dfrac{27}{36} = \dfrac{3}{4}$

CAREFUL – do not reduce until after the power rule is applied!!

TRY: $(a^2b^2)^2$ $(2x^4y^2)^3$ $(2x^4)^3(y^2)^4$

Quotient and Power of a Quotient Rules

Quotient Rule for Exponents

If *m* and *n* are any integers and $a \neq 0$, then $a^m \div a^n = \dfrac{a^m}{a^n} = a^{m-n}$

To divide expressions with like bases, subtract the exponent of the denominator from the exponent of the numerator to get the exponent of the common base in the quotient.

$$\frac{a^5}{a^2} = a^{5-2} = a^3 \qquad \frac{y^{-3}}{y^{-5}} = y^{-3--5} = y^2$$

TRY: $\dfrac{a^7}{a^4}$

Power of a Quotient Rule

If *a* and *b* are nonzero real numbers and *n* is any integer, then $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

When a fraction is raised to a power, the numerator and the denominator are both raised to that power.

$$\left(\frac{5x}{2y^4}\right)^3 = \frac{5^3x^3}{2^3y^{12}} = \frac{125x^3}{8y^{12}} \qquad \left(\frac{2}{3}x\right)^3 = \left(\frac{2}{3}\right)^3 x^3 = \frac{8}{27}x^3$$

TRY: $\left(\dfrac{6m}{7p}\right)^2$ $\qquad \dfrac{(3a^2)^3}{(4a^3)^2}$

Quick Review of Exponent Rules

Review: Think about the following: a^3 What does it mean? In the **exponential** expression a^3 (read "a to the third power"), **a** is called the **base** and **3** is called the **exponent.**

Definition: If *a* is a nonzero real number and *n* is a positive integer, then $a^n = a \cdot a \cdot a \cdot ... \cdot a$

$$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$$

TRY: 4^3

Be careful: Expressions of the form $(-2)^4$ and -2^4

These expressions are not always equal.

$(-2)^4 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) = 16$ $\qquad -2^4 = -2 \cdot 2 \cdot 2 \cdot 2 = -16$

The placement of the ( ) makes a difference.

TRY: $(-3)^2$ and -3^2

Product Property for Exponents

If *m* and *n* are integers and *a* ≠ 0, then, $a^m \cdot a^n = a^{m+n}$

i.e. When multiplying expressions with **like bases**, add the exponents to get the exponent of the common base. If the base is a number, remember NOT to multiply the base.

$$3^4 \bullet 3^2 = 3^{4+2} = 3^6 = 729 \qquad -2^2 y^2 \bullet 5y^4 = -20y^6$$

Since the bases are different in $x^4 \cdot y^2 = x^4 y^2$, the factors cannot be combined.

TRY: $2^3 \bullet 2^4$ $\qquad x^5 \bullet 5x^7$ $\qquad 4x^3 \bullet 5x$

Power Property of Exponents

If *m* and *n* are any integers and *a* ≠ 0, then $(a^m)^n = a^{m \bullet n}$

i.e. A power to a power is found by multiplying the exponents.

$$(2^3)^2 = 2^{3\bullet 2} = 2^6 = 64 \qquad (2x^2)^4 = 2^{1\cdot 4} x^{2\cdot 4} = 2^4 x^8 = 16x^8 \qquad \frac{(3a^2)^3}{(6a^3)^2} = \frac{3^3 a^6}{6^2 a^6} = \frac{3^3}{6^2} = \frac{27}{36} = \frac{3}{4}$$

CAREFUL – do not reduce until after the power rule is applied!!

TRY: $(a^2b^2)^2$ $\qquad (2x^4y^2)^3$ $\qquad (2x^4)^3(y^2)^4$

Power of a Product Property of Exponents

If *a* and *b* are nonzero real numbers and *n* is any integer, then $(ab)^n = a^n b^n$

i.e. When a group of factors is raised to a power, raise each of the **factors** in the group to this power.

WARNING:

$(xy)^5 = x^5y^5$ and $(2ab)^4 = 2^4a^4b^4$ but $(2+3)^3 \neq 2^3 + 3^3$ because 2 and 3 are terms, NOT factors!

$(2+3)^3 = (5)^3$

$(-2x)^2 = (-2)^2(x)^2 = 4x^2$ This is different than $-2^2x^2 = -4x^2$

TRY: $(3y)^3$ $(-3y)^3$ $-(3y)^3$

$4(3y)^3$ $-2(-3y)^3$ $-(2xy)^3$

Quotient Properties of Exponents

If m and n are any integers with $m > n$ and $b \neq 0$, then $\frac{b^m}{b^n} = b^{m-n}$ and $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

To divide expressions with like bases, subtract the exponent of the denominator from the exponent of the numerator to get the exponent of the common base in the quotient.

$\frac{a^5}{a^2} = a^{5-2} = a^3$ $\left(\frac{x}{y^4}\right)^3 = \frac{x^3}{y^{12}}$

TRY: $\frac{3^6}{3^4}$ $\frac{a^7}{a^4}$ $\left(\frac{6}{7}\right)^2$

TRY: A Quick Review

1) 6^2 __________

2) 7^2 __________

3) 3^2 __________

4) 5^2 __________

5) 9^2 __________

6) 2^3 __________

7) -3^2 __________

8) 8^2 __________

9) 6^0 __________

10) 10^2 __________

11) 1^5 __________

12) -5^2 __________

13) 3^3 __________

14) $(-2)^2$ __________

15) 2^3 __________

16) $(-1)^5$ __________

17) 10^3 __________

18) $(-3)^2$ __________

19) -1^6 __________

20) 12^2 __________

21) 2^4 __________

22) 8^0 __________

23) $(-4)^2$ __________

24) 11^2 __________

25) -7^2 __________

26) $(-2)^3$ __________

27) 4^2 __________

28) -3^2 __________

29) 12^0 __________

30) -6^2 __________

Zero and Negative Exponent Rules

Zero Exponent Rule

If *a* is any nonzero real number then, $a^0 = 1$

Any number other than 0 raised to the zero power is equal to 1.

Consider: $\dfrac{a^5}{a^5} = a^{5-5} = a^0 = 1$

$3^0 = 1$ $(-5)^0 = 1$ $-(5)^0 = -1$ $5y^2z^{-1}(y^{-2}z) = 5y^0z^0 = 5\cdot1\cdot1 = 5$

TRY: 9^0 $4x^2(2x^{-2})$ $\dfrac{2x^{-7}}{x^{-7}}$ $-(3^0)$

Negative Exponents

Consider:

Using the Power of a Quotient Rule: $\dfrac{a^3}{a^5} = a^{3-5} = a^{-2}$

One can think of it as: $\dfrac{a\cdot a\cdot a}{a\cdot a\cdot a\cdot a\cdot a} = \dfrac{1}{a\cdot a} = \dfrac{1}{a^2}$ or $a^{-2} = \dfrac{1}{a^2}$

Negative Exponent Rule

For all real numbers a, a ≠ 0, and positive integer *n*, $a^{-n} = \dfrac{1}{a^n}$ ex: $4^{-2} = \dfrac{1}{4^2} = \dfrac{1}{16}$

If *a* and *b* are nonzero real numbers and *n* is a positive integer then,

$$a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n} \qquad a^{-1} = \frac{1}{a} \qquad \frac{1}{a^{-n}} = a^n$$

A negative exponent on any base (except 0) can be written as one over that base with a positive exponent. Thus, if a factor is moved either from the numerator to the denominator or from the denominator to the numerator, the sign of its exponent will change. The sign of the base will not be affected by the change.

$x^{-2} = \frac{1}{x^2}$ $\quad 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ $\quad \frac{1}{x^{-3}} = x^3$ $\quad \frac{1}{4^{-2}} = 4^2 = 16$ $\quad (x^2)^{-4} = x^{-8} = \frac{1}{x^8}$

TRY:

m^{-4} $\quad 2^{-3}$ $\quad \frac{1}{x^{-4}}$ $\quad \frac{1}{5^{-2}}$ $\quad -2^{-3}$

Another way of thinking: When a base and exponent are moved across the division bar (from the numerator to the denominator), the sign on the exponent changes.

$\frac{a^5b^{-3}}{a^{-3}b^2} = \frac{a^5a^3}{b^2b^3} = \frac{a^8}{b^5}$ Move the bases with negative exponents, then simplify.

TRY:

$\frac{a^{-5}b^3}{a^4b^{-2}}$ $\quad \frac{y^{-5}}{y^{-2}}$ $\quad \frac{5z^{-3}}{a^2 3^{-2}}$

Careful – when moving bases, only change the sign of the exponent: $(-2)^{-2} = \frac{1}{(-2)^2} = \frac{1}{4}$

When a fraction is raised to a negative exponent, write its reciprocal with a positive exponent.

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n} = \frac{b^n}{a^n}$$

$$\left(\frac{4}{5}\right)^{-3} = \left(\frac{5}{4}\right)^{3} = \frac{5^3}{4^3} = \frac{125}{64}$$

TRY:

$$\left(\frac{2}{3}\right)^{-3}$$

Be careful, DO NOT change any of the signs of exponents within ( ).

$$\left(\frac{2a^{-2}b}{3}\right)^{-3} = \left(\frac{3}{2a^{-2}b}\right)^{3} = \frac{3^3}{2^3a^{-6}b^3} = \frac{3^3a^6}{2^3b^3} = \frac{27a^6}{8b^3}$$

Another way of thinking: Sometimes it is easier to use the Power of a Power Rule first and then work with any negative exponents. Many students find using this approach helps them avoid the error of switching the sign of exponents within ( ) as well as the sign of the exponent on the outside ( ).

$$\left(\frac{2a^{-2}b}{3}\right)^{-3} = \frac{2^{-3}a^6b^{-3}}{3^{-3}} = \frac{3^3a^6}{2^3b^3} = \frac{27a^6}{8b^3}$$

Before trying some more problems, think about how to solve this problem: $3^{2n-3} \cdot 3^{4-2n}$

What does one do with exponents when like bases are multiplied? Do it.

Using Exponent Rules

TRY: evaluate each of the following expressions. Leave no negative exponents.

$(2x^{-1}y^{2})^{-3}$

$-2y^{-3}(-5y^{-4})$

$\dfrac{(a^{2})^{-3}}{(a^{-2})^{4}}$

$\dfrac{-3y(3y^{-3})}{6y^{-2}}$

$(x^{-2})^{3}(x^{-3})^{-2}$

$\left(\dfrac{ab^{-3}}{a^{2}b}\right)^{-2}$

$-3a^{5}b^{-6}\cdot 2a^{-5}b^{2}$

$\dfrac{-3y^{-3}}{y^{-2}z^{3}}$