Zero and Negative Exponent Rules

Zero Exponent Rule

If *a* is any nonzero real number then, $a^0 = 1$

Any number other than 0 raised to the zero power is equal to 1.

Consider: $\frac{a^5}{a^5} = a^{5-5} = a^0 = 1$

$3^0 = 1$ $(-5)^0 = 1$ $-(5)^0 = -1$ $5y^2z^{-1}(y^{-2}z) = 5y^0z^0 = 5 \cdot 1 \cdot 1 = 5$

TRY: 9^0 $4x^2(2x^{-2})$ $\frac{2x^{-7}}{x^{-7}}$ $-(3^0)$

Negative Exponents

Consider:

Using the Power of a Quotient Rule: $\frac{a^3}{a^5} = a^{3-5} = a^{-2}$

One can think of it as: $\frac{a \cdot a \cdot a}{a \cdot a \cdot a \cdot a \cdot a} = \frac{1}{a \cdot a} = \frac{1}{a^2}$ or $a^{-2} = \frac{1}{a^2}$

Negative Exponent Rule

For all real numbers a, a ≠ 0, and positive integer *n*, $a^{-n} = \frac{1}{a^n}$ ex: $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

If *a* and *b* are nonzero real numbers and *n* is a positive integer then,

$$a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n} \qquad a^{-1} = \frac{1}{a} \qquad \frac{1}{a^{-n}} = a^n$$

A negative exponent on any base (except 0) can be written as one over that base with a positive exponent. Thus, if a factor is moved either from the numerator to the denominator or from the denominator to the numerator, the sign of its exponent <u>will change</u>. The sign of the base <u>will not</u> be affected by the change.

$$x^{-2} = \frac{1}{x^2} \qquad 3^{-2} = \frac{1}{3^2} = \frac{1}{9} \qquad \frac{1}{x^{-3}} = x^3 \qquad \frac{1}{4^{-2}} = 4^2 = 16 \qquad (x^2)^{-4} = x^{-8} = \frac{1}{x^8}$$

TRY:

$$m^{-4} \qquad 2^{-3} \qquad \frac{1}{x^{-4}} \qquad \frac{1}{5^{-2}} \qquad -2^{-3}$$

Another way of thinking: When a base and exponent are moved across the division bar (from the numerator to the denominator), the sign on the exponent changes.

$\dfrac{a^5b^{-3}}{a^{-3}b^2} = \dfrac{a^5a^3}{b^2b^3} = \dfrac{a^8}{b^5}$ Move the bases with negative exponents, then simplify.

TRY:

$$\frac{a^{-5}b^3}{a^4b^{-2}} \qquad \frac{y^{-5}}{y^{-2}} \qquad \frac{5z^{-3}}{a^2 3^{-2}}$$

Careful – when moving bases, only change the sign of the exponent: $(-2)^{-2} = \dfrac{1}{(-2)^2} = \dfrac{1}{4}$

When a fraction is raised to a negative exponent, write its reciprocal with a positive exponent.

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n} = \frac{b^n}{a^n}$$

$$\left(\frac{4}{5}\right)^{-3} = \left(\frac{5}{4}\right)^{3} = \frac{5^3}{4^3} = \frac{125}{64}$$

TRY:

$$\left(\frac{2}{3}\right)^{-3}$$

Be careful, DO NOT change any of the signs of exponents within ( ).

$$\left(\frac{2a^{-2}b}{3}\right)^{-3} = \left(\frac{3}{2a^{-2}b}\right)^{3} = \frac{3^3}{2^3 a^{-6} b^3} = \frac{3^3 a^6}{2^3 b^3} = \frac{27a^6}{8b^3}$$

Another way of thinking: Sometimes it is easier to use the Power of a Power Rule first and then work with any negative exponents. Many students find using this approach helps them avoid the error of switching the sign of exponents within ( ) as well as the sign of the exponent on the outside ( ).

$$\left(\frac{2a^{-2}b}{3}\right)^{-3} = \frac{2^{-3} a^6 b^{-3}}{3^{-3}} = \frac{3^3 a^6}{2^3 b^3} = \frac{27a^6}{8b^3}$$

Before trying some more problems, think about how to solve this problem: $3^{2n-3} \cdot 3^{4-2n}$

What does one do with exponents when like bases are multiplied? Do it.