Lesson 11: Relations, Functions, Linear Systems

Relations

Consider the linear equation $y = 3x + 2$.

When an 'x' is selected, it is 'plugged' into the equation and the results is a 'y'.
A set of ordered pairs that make this equation true is { (0,2), (1,5), (-1,-1)}

These ordered pairs form a **relationship**. The 1st value is called the **domain**. The 2nd value is called the **range**. A special type of relation is called a **function** if each element of the domain corresponds to exactly one element of the range.

Relation

Any set of **ordered pairs** is a **relation**.

1. Complete the table #1.

x	0	1	2	3	4	25		x
y	0	2	4	6			60	2x

Based on the table data, complete the set of ordered pairs:

{ (0,0), (1,2), (2,4), (3,6), (4,), (25,), (, 60) }

2. Complete the table #2.

x	0	1	2	3	4	10		x
y	1	4	7		13		22	

Based on the table data, complete the set of ordered pairs:

{ (0,1), (1,4), (2,7), (3,), (4, 13), (10,), (, 22) }

3. Complete the table #3.

Day	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Letter	S	M	T	W	T	F	S

Based on the table data, complete both sets of ordered pairs:

Relation (day, letter)

{ (Sunday, S), (Monday, M), (Tuesday, T), (Wednesday, W), (Thursday, T), (Friday, F), (Saturday, S)}

Relation (letter, day)

{ (S, Sunday), (M, Monday), (T, Tuesday), (W, Wednesday), (T, Thursday), (F, Friday), (S, Saturday)}

All of the above are examples of RELATIONS.

Functions

Function

If the value of the variable *y* is determined by the value of the variable *x,* then *y* is a **function of *x***. There is **only one** *y* for any *x*. A **function** is a set of ordered pairs (a relation) in which no two ordered pairs have the same first coordinate and different second coordinates.

Given (x,y): the variable corresponding to the first coordinate, the x, is the **independent** variable and the variable corresponding to the second coordinate, the y, is the **dependent** variable.

Another way of thinking:

Consider the relation: (day, letter) where day is the 'x', letter is the 'y'

{ (Sunday, S), (Monday, M), (Tuesday, T), (Wednesday, W), (Thursday, T), (Friday, F), (Saturday, S)}

If you were asked what y-value goes with the x-value of Tuesday,
do you know exactly what the answer is?

YES – therefore, this set is a function even though both Tuesday and Thursday have y-values of T.

Consider the relation: (letter, day) where letter is the 'x', day is the 'y'

{ (S, Sunday), (M, Monday), (T, Tuesday), (W, Wednesday), (T, Thursday), (F, Friday), (S, Saturday)}

If you were asked what y-value goes with the x-value "S", do you know exactly what the answer is?

NO. It could be either Sunday or Saturday. Therefore this relation is NOT a function.

Two different values of x can map to the same value of y and it is still a function of x.

If one value of x maps to two different values of y, it is NOT a function of x.

Is ‘y’ a function of ‘x’?

Consider all possible rectangles. Let ‘y’ represent the area of a rectangle and ‘x’ represent the width.

x (width)	4	4
y (area)	20	???

Is ‘y’ a function of ‘x’? (i.e., Is area a function of width? For any width, will there be only one area?)

Area is actually a function of length and width. Since there are multiple y values possible for any given x, this example is not a function.

Consider all pizzas sold at Pizzaz with a 6% sales tax.
Let ‘y’ represent the amount of sales tax and ‘x’ represent the original price of the pizza.

x (original price)	10	10
y (sales tax at 6%)	0.60	????

Is ‘y’ a function of ‘x’? (i.e., Is sales tax a function of the original price? For any original price, will there be only one amount of sales tax?) Given the x, there WILL BE exactly one y in this example. This is a function.

TRY:

Consider the set: { (0,3), (2, 5), (4, 7), (2, -5), (4, -7)}

If you were given this set and asked what y goes with the x-value of 4, do you know exactly what the answer is? Is this a function?

Consider the set: { (3,0), (5, 2), (7, 4), (-5, 2), (-7, 4)}

If you were given this set and asked what y goes with the x-value of 5, do you know exactly what the answer is? Is this a function?

Vertical Line Test

Vertical Line Test

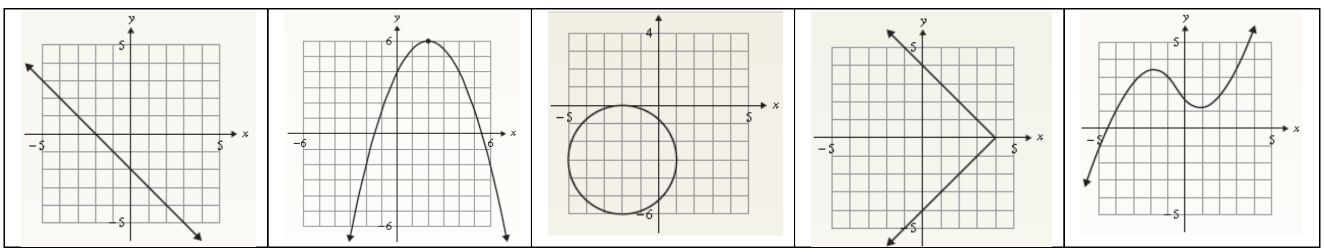
A graph is the graph of a function if and only if there is no **vertical** line that crosses the graph more than once.

Does the following represent a function?

{ (-4, 2), (-2, 0), (0, -2), (2, -4) }   These points are on the line of the first graph on the next page. Any vertical line drawn through the line of the graph does not cross in more than one place. Therefore, the graph does represent a function.

{ (-5, -3), (-2, 0), (1, -3), (-2, -6) } These points are on the line of the third graph on the next page. Any vertical line drawn through the circle of the graph cross in more than one place. Therefore, the graph does NOT represent a function.

Consider the other three graphs – which represent functions, which do not? Why?



Consider the 1st graph above. The domain is any x value along the real number line. In fact, the domain is all the real numbers from negative infinity to positive infinity.

This is written as the interval: $(-\infty, \infty)$

Which of the following relations (given as lists of ordered pairs) represent function?

Plot them and use the vertical line test if desired.

{(1,3), (1,5), (3,6)}   {(4,6), (5,7), (6,8)}

Domain and Range

Domain and Range of a Relation or Function

1. The **domain** of a relation or function is the set of first components of the ordered pairs.

2. The **range** of a relation or function is the set of second components of the ordered pairs.

Note – if the domain is not stated, assume the domain to be the set of all those real numbers for which the function is defined, or makes sense (that is, when the values of the domain are substituted for the independent variable, they produce real numbers for the range, the dependent variable).

What is the domain and range of the function represented by:
{ (Sunday, S), (Monday, M), (Tuesday, T), (Wednesday, W), (Thursday, T), (Friday, F), (Saturday, S)}

Domain: Range:

What is the domain and range of the function represented by {(0,0), (1,2), (2,4), (3,6)} ?

Domain: Range:

Which of the following relations (given as equations) represent functions?

What is the domain? Is it a function?

Determine some points, plot them, and use the vertical line test if desired.

$y^2 = x - 6$	$y = 4x - 3$	$y = \|x - 1\|$	$y = \dfrac{6}{x-2}$
Domain: $[6, \infty)$	Domain:	Domain:	Domain: $(-\infty, 2) \cup (2, \infty)$
If x is 15, then y^2 must be 9. What value(s) of y result in 9? Can the same x map to two different y values and it still be a function?	Function?	If x is 4, y is 3. If x is -2, y is 3. Can different x values map to the same y value and it still be a function?	Since 2 would cause the denominator to be 0 and division by 0 is undefined, 2 must not be included in the domain.

Function Notation

If y is a function of x, **function notation** f(x) is used to represent y.
In Table #2, $y = 3x + 1$. Using function notation it would be written: $f(x) = 3x + 1$

Write $y = 3x - 2$in function notation:

Definition of a Linear Function

A function of the form f(x) = *m*x + b is called a **linear function** if *m* ≠ 0 and is called a constant function if *m* = 0 where *m* and b are real numbers, and *m* is the slope and the point (0,b) is the y-intercept. [Meaning: the function, when graphed, results in a straight line.

Working with functional notation

Consider the following problem. Given the function $f(x) = 3x + 1$, find $f(4)$. This is another way to ask you to find the value of y, using 4 as the x.

In the past, you have been asked to substitute the value of 4 for x in the equation. In this example, $f(4) = 13$since 3(4) +1 = 13.

Let $f(x) = 3x - 2$ and $g(x) = x^2 - x$ What is $f(4)$?

What is $g(-3)$? What is $f(0)$ + $g(4)$?

CAUTION: Sometimes a problem is reversed. Given the function $f(x) = 3x + 1$, find x when $f(x) = 10$. This is asking: what value of x is used to get a result of 10? If 3 is used for x, the result is 10. For this type of problem, set $3x + 1 = 10$ and solve for x.

Given $f(x) = 3x - 2$, find x when $f(x) = 13$

Solving Linear Systems of Equations by Graphing

Consider the equations: $y = x + 1$ and $x + 2y = 8$

What (x,y) point satisfies both equations?

System of Linear Equations

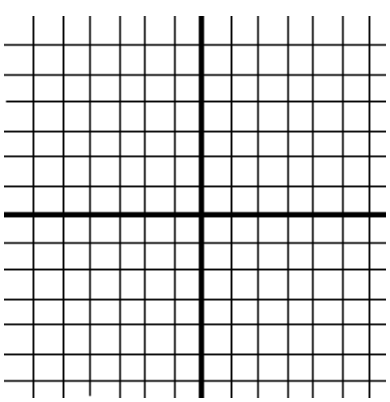
Two or more linear equations involving the same variables form a system of linear equations. A solution of this system is any ordered pair that satisfies *both* equations – called a solution set of the system.

There are three ways to find the solution: (1) by graphing, (2) by substitution, and (3) by elimination.

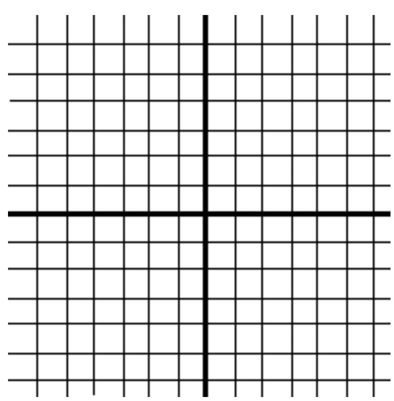
Graphs of Systems of Linear Equations – three possibilities

1. The *graphs intersect in a single point*. The solution is the point of intersection. This system is called consistent and **independent**.
   Solution: { (1, 3) }
2. The *graphs are parallel lines*. There is no solution and the system is called **inconsistent**.
   Solution: $\varnothing$
3. The *graph is the same line*. Any solution of one equation is a solution of the other equation. The system is called **dependent**.
   Solution: { (x,y) | x-2y=4 }

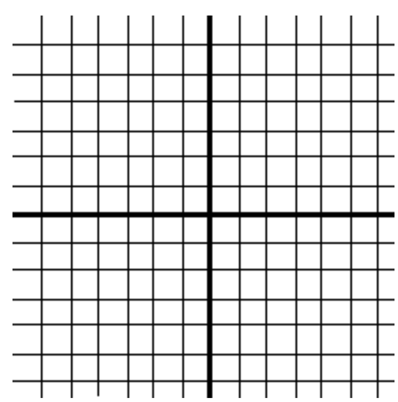
Graph: $y = x + 2$
$x + y = 4$



Graph: $2x – 3y = 6$
$3y – 2x = 3$

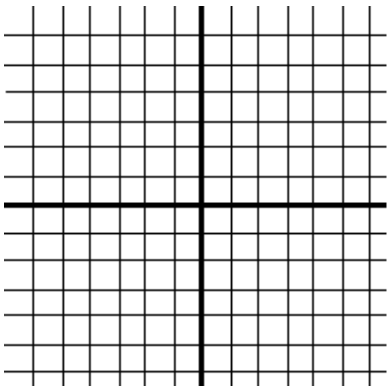


Graph: $2(y + 2) = x$
$x – 2y = 4$



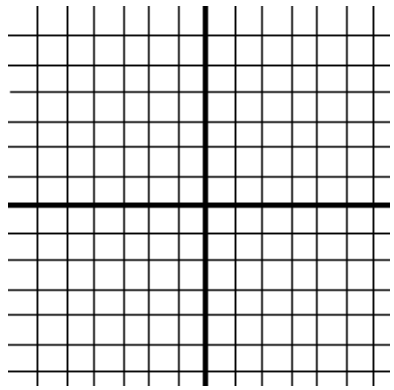
Solve by graphing:

y = x – 3
y = – x + 1



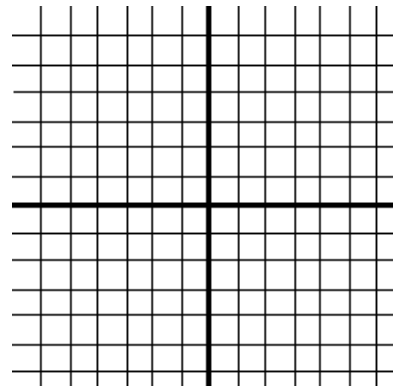
Solve by graphing:

y = 2x + 1
x + y = – 2



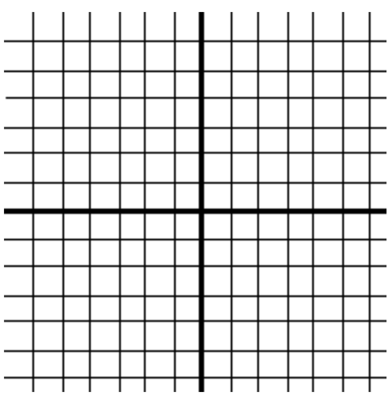
Solve by graphing:

y = – 3x
x + y = 2



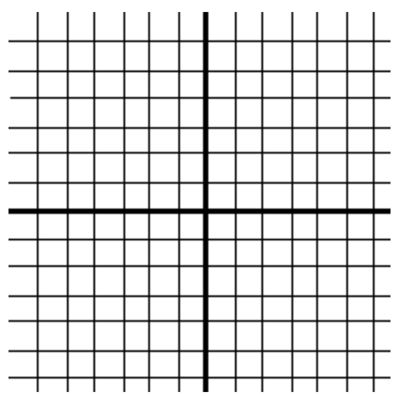
Solve by graphing:

3y – 3x = 9
x – y = 1



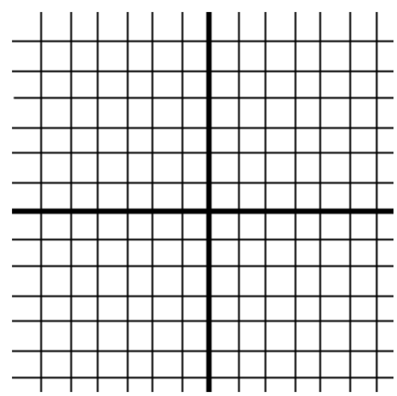
Solve by graphing:

2x – 3y = 6
y = 2/3 x – 2



Solve by graphing:

3x – 2y = 6
3x + 2y = 6



What would be one of the difficulties of using the graphing method?

Solving Linear Systems of Equations by Substitution

To Solve Systems of Two Linear Equations by Substitution

1. Solve one of the equations for one of the variables. (If one of the variables has a coefficient of 1 or –1, solve for it.)

2. Substitute the expression obtained in step 1 for that variable in the other equation, and solve the resulting equation in one variable.

   a. If this results in a value for one of the variables, substitute the value obtained for the variable into one of the *original* equations and solve for the other variable. There is one solution.

   b. If this results in the variables being eliminated and a true statement obtained, there are infinitely **many solutions** and the system is **dependent** (same line).

   c. If this results in the variables being eliminated and a false statement obtained, there is **no solution** and the system is **inconsistent** (parallel lines).

3. If the system has a solution is obtained in step 2a, check the solution in both equations.

Example: Solve by substitution $2x + 3y = 14$
$y = 3x + 1$

The second equation has already been solved for y.

Use this value, $3x + 1$, to substitute for y in the first equation.

$2x + 3(3x+1) = 14$

Now solve for x.

$2x + 9x + 3 = 14$
$11x + 3 = 14$
$11x = 11$
$x = 1$ Now, substitute the value of 1 in for x in one of the equations.

$y = 3(1) + 1$
$y = 4$ Check the solution $\{(1,4)\}$ in both of the original equations.

Solve by substitution:

$y = x + 2$
$x + y = 4$

$2x - 3y = 6$
$3y - 2x = 3$

$2(y + 2) = x$
$x - 2y = 4$

TRY:

$y = x + 4$
$3y - 5x = 6$

$x = y + 3$
$3x - 2y = 4$

$3y - x = 0$
$x - 4y = -2$

$2x - y = 4$
$2x - y = 3$

Solving Linear Systems of Equations by Elimination

To Solve a System of Two Linear Equations by the Elimination Method:

1. Write the system so that each equation is in standard form $Ax + By = C$.

2. Multiply one equation (or both equations), if necessary, by a number to obtain *additive inverse coefficients* for one of the variables.

3. **Add** the corresponding sides of the resulting equations and solve the resulting equation in *one variable*.

    a. If this results in a value for one of the variables, substitute the value obtained for the variable into one of the *original* equations and solve for the other variable. There is one solution.

    b. If this results in both variables being eliminated and a **false statement** is obtained, there is **no solution** and the system is **inconsistent** (parallel lines).

    c. If this results in both variables being eliminated and a **true statement** such as 0 = 0 is obtained, there are infinitely **many solutions** and the system is **dependent** (same line).

4. If the system has a solution found in step 3a, check the solution in both equations.

Consider: $2x - y = 5$ and $-x + y = -6$

Remember, the addition property of equality says that one can add something of equal value to both sides of an equation without changing the equality. This property allows one to add these two equations together.

$$\begin{array}{l} 2x - y = 5 \\ \underline{-x + y = -6} \\ x \qquad = -1 \end{array}$$

Now, substitute -1 in either equation for x and solve for y.

$2(-1) - y = 5$ … $y = -7$ Solution: $\{ (-1, -7) \}$

Consider: $x + 3y = -12$ Multiple each term in the 1st equation by -3. $-3x - 9y = 36$

$3x + 4y = -6$ Now add the 2nd. $3x + 4y = -6$ Continue….

Consider: $2x + 3y = 6$ To eliminate the y, multiply the 1st by 2 ➔ $4x + 6y = 12$

$5x + 2y = -7$ Multiply the 2nd by -3 ➔ $-15x - 6y = 21$ Continue…

Solve by elimination:

$y = x + 2$
$x + y = 4$

$2x - 3y = 6$
$3y - 2x = 3$

$2(y + 2) = x$
$x - 2y = 4$

TRY:

$3x + 5y = -11$
$x - 2y = 11$

$x - y = 3$
$-6x + 6y = 17$

$2x = 2 - y$
$3x + y = -1$

$3x - 4y = 11$
$-3x + 2y = -7$