

Solving Linear Systems of Equations by Elimination

To Solve a System of Two Linear Equations by the Elimination Method:

1. Write the system so that each equation is in standard form $Ax + By = C$.
2. Multiply one equation (or both equations), if necessary, by a number to obtain *additive inverse coefficients* for one of the variables.
3. **Add** the corresponding sides of the resulting equations and solve the resulting equation in *one variable*.
 - a. If this results in a value for one of the variables, substitute the value obtained for the variable into one of the *original* equations and solve for the other variable. There is one solution.
 - b. If this results in both variables being eliminated and a **false statement** is obtained, there is **no solution** and the system is **inconsistent** (parallel lines).
 - c. If this results in both variables being eliminated and a **true statement** such as $0 = 0$ is obtained, there are infinitely **many solutions** and the system is **dependent** (same line).
4. If the system has a solution found in step 3a, check the solution in both equations.

Consider: $2x - y = 5$ and $-x + y = -6$

Remember, the addition property of equality says that one can add something of equal value to both sides of an equation without changing the equality. This property allows one to add these two equations together.

$$2x - y = 5$$

$$\underline{-x + y = -6}$$

$x = -1$ Now, substitute -1 in either equation for x and solve for y .

$$2(-1) - y = 5 \quad \dots \quad y = -7 \quad \text{Solution: } \{(-1, -7)\}$$

Consider: $x + 3y = -12$ Multiple each term in the 1st equation by -3 . $-3x - 9y = 36$
 $3x + 4y = -6$ Now add the 2nd. $3x + 4y = -6$ Continue....

Consider: $2x + 3y = 6$ To eliminate the y , multiply the 1st by 2 → $4x + 6y = 12$
 $5x + 2y = -7$ Multiply the 2nd by -3 → $-15x - 6y = 21$ Continue...

Solve by elimination: $y = x + 2$ $2x - 3y = 6$ $2(y + 2) = x$
 $x + y = 4$ $3y - 2x = 3$ $x - 2y = 4$

TRY:

$$\begin{aligned} 3x + 5y &= -11 \\ x - 2y &= 11 \end{aligned}$$

$$\begin{aligned} x - y &= 3 \\ -6x + 6y &= 17 \end{aligned}$$

$$\begin{aligned} 2x &= 2 - y \\ 3x + y &= -1 \end{aligned}$$

$$\begin{aligned} 3x - 4y &= 11 \\ -3x + 2y &= -7 \end{aligned}$$