Slope

The slope of a line is the "slant" or "steepness" of the line.
Definition: If $x_1 \neq x_2$, the **slope** (m) of the line containing points (x_1, y_1) and (x_2, y_2) is defined by

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad m = \frac{\text{Rise}}{\text{Run}} \quad \text{or} \quad m = \frac{\text{the vertical change in the y-coordinate}}{\text{the horizontal change in the x-coordinate}}$$

Note: The x- and y-values may be subtracted in any order so long as the coordinates of each point are in the same position in the numerator and the denominator.

The Slope of a Line Is …

- **Positive** if the line slants up from left to right. As the value of x increases, the value of y increases.
- **Negative** if the line slants down from left to right. As the value of x increases, the value of y decreases.
- **Zero** if the line is horizontal (parallel to the x-axis).
- **Undefined** if the line is vertical (parallel to the y-axis).

What is the slope of:

1) a line through (2, 5) and (6, 3)

$$m = \frac{5-3}{2-6} = \frac{2}{-4} = -\frac{1}{2}$$

Let (2,5) be point 1 and (6,3) be point 2. It doesn't matter which one uses for point 1. What is important is that one uses the same point for x_1 and y_1 and the same point for x_2 and y_2.

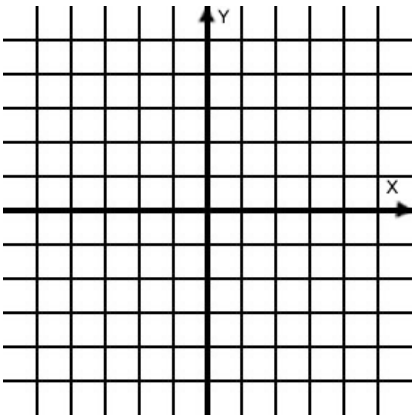
With a slope of $-\frac{1}{2}$, the line slopes downward.
It has a negative slope.

Another point would be "down 1, over 2" from (2, 5).
[Think of – ½ as -1 / 2.]

This means the change in Y will be -1 and the change in X will be +2 resulting in the point (2 + 2 , 5 - 1) or (4, 4).

Another point would be "down 1, over 2" or (4+2, 4-1) or (6,3) – the second point given.

Plot the original two points and draw the line.
Is the point (4, 4) included?



2) a line through (-2, 3) and (-5,-1)

$$m = \frac{3-(-1)}{-2-(-5)} = \frac{4}{3}$$

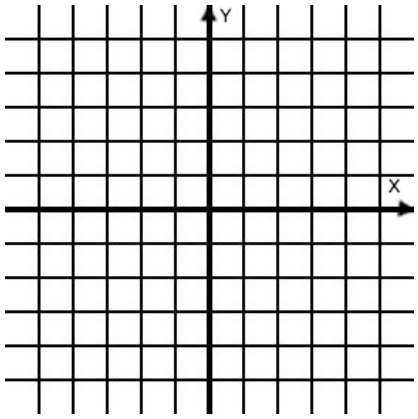
slopes upward; has a positive slope;
another point "up four, over 3" or (1,7)

CAUTION: It is very easy to make an error and use the wrong x or y value for finding another point. Be careful.

$$(-2,3)$$

Sometimes it help to write it as: $\dfrac{+3,+4}{(1,7)}$

Plot the three points. Graph the line.

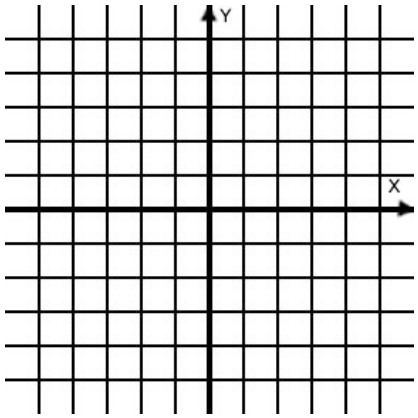


3) a line through (-6, 4) and the origin

$$m = \frac{4-0}{-6-0} = \frac{4}{-6} = -\frac{2}{3}$$

Slopes downward, negative
another point "down 2, over 3"
(-6 +3, 4 -2) or (-3, 2)

Plot the points. Graph the line.

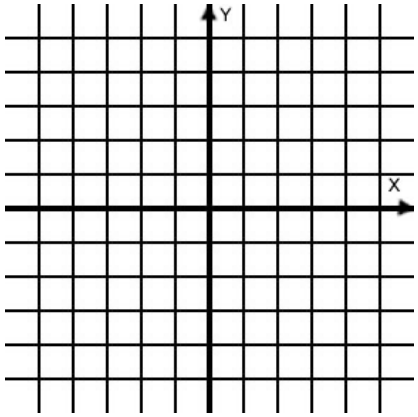


4) a line through (1,2) and (1,-3)

$$m = \frac{2-(-3)}{1-1} = \frac{5}{0}$$ Slope is UNDEFINED.

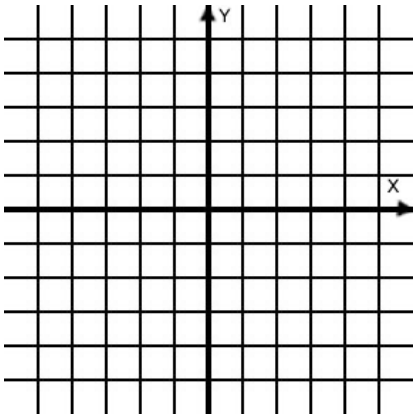
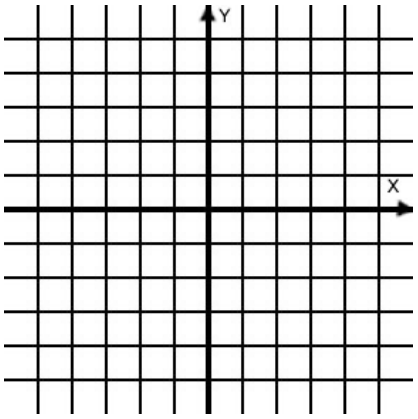
This is a VERTICAL line crossing the x-axis at $x = 1$.
Graph the line.

What is another point on the line?



Note:
Had the slope of the problem had 0 for the numerator instead of the denominator, the slope would have been 0 and the line would have been a HORIZONTAL line crossing the y-axis.

TRY:

5) a line through (-3, 2) and (-1, 5) Slope? Direction of slope? Another point? Plot the three points. Graph the line.	
6) a line through the origin and (3, -2) Slope? Direction of slope? Another point? Plot the three points. Graph the line.	
7) a line through (-2, 2) and (3, 2) Slope? Direction of slope? Another point? Plot the three points. Graph the line.