Lesson 08: Proportions and Percents

Proportions

Vocabulary

Proportion	An equation that compares two equal fractions (or rates) Ex: $\frac{3}{5}=\frac{6}{10}$ is read "3 is to 5 as 6 is to 10"

TRY: Write the proportions.

5 is to 18 as 15 is to 54 $\frac{\square}{\square}=\frac{\square}{\square}$

4 is to 12 as 20 is to 60 $\frac{\square}{\square}=\frac{\square}{\square}$

Is this the same as:
If it takes 4 minutes to fold 12 books, working at the same rate, it will take 20 minutes to fold 60 books.

A proportion is known to be true if the **cross product**,
the product of the **extremes** equals the product of the **means**.

That is, $\frac{3}{5}=\frac{6}{10}$ when $3\cdot 10=5\cdot 6$

$extremes = means$

Extremes–Means Property.

If $\frac{a}{b}=\frac{c}{d}$, then $ad=bc$ provided that $b\neq 0$ and $d\neq 0$.

TRY: Determine whether each pair of fractions is equivalent.

$\frac{3}{7}=\frac{27}{63}$ $\frac{4}{5}=\frac{3}{4}$ $\frac{\frac{5}{1}}{\frac{}{4}}=\frac{200}{10}$

TRY: Use the extremes = means rule to find the value of x.

$\frac{6}{7}=\frac{x}{56}$ $\frac{3}{8}=\frac{5}{x}$ $\frac{\frac{2}{5}}{9}=\frac{10}{x}$ $\frac{0.8}{x}=\frac{5}{40}$

Proportion Problems

If 5 feet of rope costs $2.10, what would 7 feet of rope cost?

This problem can be thought of in two ways:

$\frac{5}{7} = \frac{2.10}{x}$ **OR** $\frac{5}{2.10} = \frac{7}{x}$ **Either way,** $5x = 7(2.10)$

TRY:

If 12 apples cost $4.80, what would 5 apples cost?

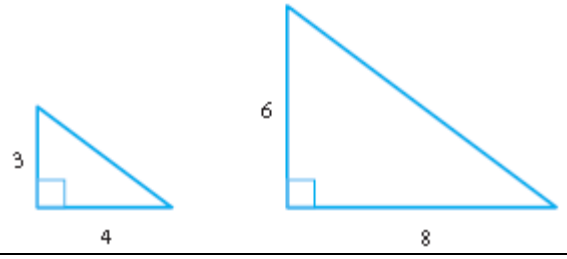
JJ worked 2.4 hours and received $8.64. If KT works 10 hours at the same rate of pay, how much will KT receive?

The paint machine can paint 30 signs in 2 minutes. How many signs can be painted in 4 hours?

Proportions: Similar Triangles

Vocabulary

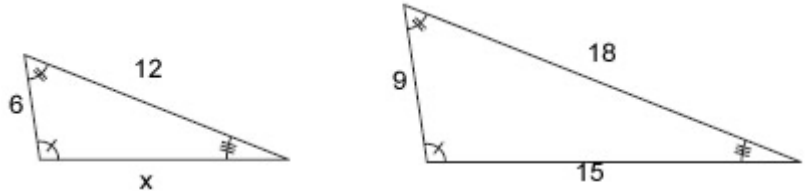
Similar	Two right triangles are similar if the ratios of corresponding sides are equivalent.



$\frac{3}{4}=\frac{6}{8}$ or 3 is to 4 as 6 is to 8

$\frac{3}{6}=\frac{4}{8}$ or 3 is to 6 as 4 is to 8

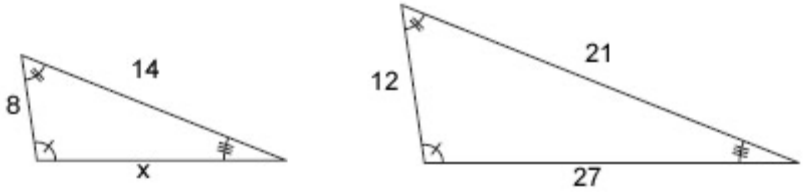
In geometry, if triangles are similar, they are proportional.



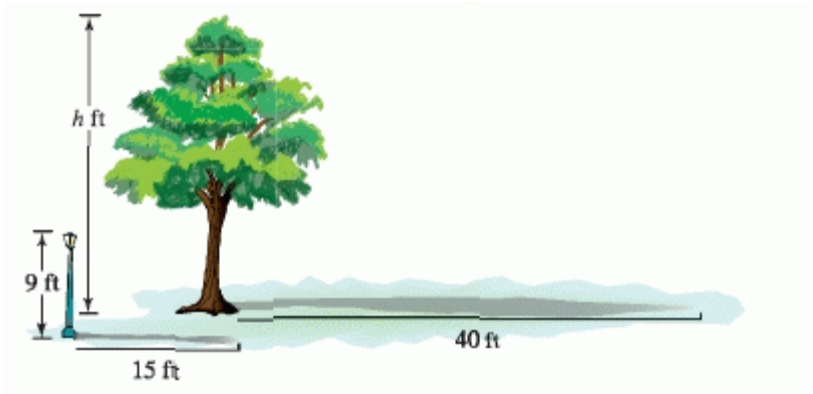
$\frac{6}{9}=\frac{x}{15}$ $\frac{12}{18}=\frac{x}{15}$ or even $\frac{6}{x}=\frac{9}{15}$

TRY:

Find the length of the unknown side x, given these two similar triangles.



A 9' lamppost casts a 15' shadow. At the same time of day, how high of a tree will cast a 40' shadow?



Percents – Decimals - Fractions

Vocabulary

Percent	Means "for each hundred"
Percent sign, %	Read "out of each 100"

Examples – using percent notation TRY:

25 out of 100 ➔ $\frac{25}{100}$ ➔ 25% 15 out of 100 ➔

35 out of 50 ➔ $\frac{35}{50}=\frac{70}{100}$ ➔ 70% 13 out of 25 ➔

Examples – changing a percent to a fraction TRY:

9% ➔ $\frac{9}{100}$ 21%

80% ➔ $\frac{80}{100}=\frac{4}{5}$ 75%

Examples – changing a percent to a mixed number TRY:

140% ➔ $\frac{140}{100}=\frac{14}{10}=\frac{7}{5}=1\frac{2}{5}$ 275% ➔

$33\frac{1}{3}\%$ ➔ $\frac{33\frac{1}{3}}{100}=\frac{\frac{100}{3}}{100}=\frac{100}{3}\bullet\frac{1}{100}=\frac{1}{3}$ $66\frac{2}{3}\%$ ➔

Examples – changing a percent to a decimal equivalent

TRY:

80% ➔ $\frac{80}{100}$ ➔ .80 or .8 30% ➔

9% ➔ $\frac{9}{100}$ ➔ .09 3% ➔

This is the same as moving the decimal point (understood or shown) in the percent to the left 2 places.

145% ➔ 1.45 425% ➔

0.55% ➔ 0.0055 0.075% ➔

(Careful – this is less than 1%,
so the decimal should be less than 1/100)

Examples – various conversions

$5\frac{3}{4}$ ➔ $\frac{23}{4}=\frac{575}{100}$ ➔ 575% ➔ 5.75

Move the decimal 2 places to the right to change from decimal to %.

0.065 ➔ $\frac{65}{1000}$ ➔ $\frac{6.5}{100}$ ➔ 6.5%

0.007 ➔ 0.7% or $\frac{7}{10}\%$

TRY:

Write as fractions or mixed number:

1. 2%
2. 37.5 %
3. 150%

Write as a decimal

1. 75%
2. 6.25%
3. 225%

Write as a percent

1. 0.06
2. 0.375
3. 2.4

Write as a percent

1. $\frac{43}{100}$
2. $\frac{2}{5}$
3. $2\frac{2}{3}$

Percents: Problems Solved with Proportions

Vocabulary

Base	The "whole" in a problem. The standard used for comparison.
Amount	The "part" of the whole being compared to the base. Amount = Rate% of Base ➔ $Amount = \frac{r}{100} \bullet Base$
Rate	The ratio of the amount to the base written as a percent.
Percent Proportion	Proportion showing the relationship of the Rate, Amount, and Base $\frac{r}{100} = \frac{Amount}{Base}$

Examples:

Problem	Setup	Answer
15% of what number is 18? Rate Base Amount	$\frac{15}{100} = \frac{18}{B}$	$15B = 1800$ $B = 120$
45 is 25% of what number Amount Rate Base	$45 = \frac{25}{100} \bullet B$ or $\frac{25}{100} = \frac{45}{B}$	$25B = 4500$ $B = 180$
What % of 200 is 84? Rate base Amount	$\frac{r}{100} = \frac{84}{200}$	$200r = 8400$ $r = 42$ Rate=42%
150% of 90 is what? Rate Base Amount	$\frac{150}{100} = \frac{A}{90}$	$13500 = 100A$ $A = 135$

Did you notice that the "of" number is always the base?

TRY: Complete the following.

80 is 4% of what number?		
70 is what percent of 50?		
11% of 3,000 is what number?		
Find the base if 12.5% of the base is 625		
90 is 120% of what number?		
24 is what percent of 192?		

Examples-

Problem	Setup	Answer
What is 6.5% of 400? Amount Rate Base	$\frac{6.5}{100}=\frac{A}{400}$	$2600=100A$ $A=26$
650 is what % of 400? Amount Rate Base	$\frac{r}{100}=\frac{650}{400}$	$400r=65000$ $r=162.5$ Rate=162.5%
Find the base if 275% of the base is 220 Rate Base Amount	$\frac{275}{100}=\frac{220}{B}$	$275B=22000$ $B=80$
What is 365% of 86? Amount Rate Base	$\frac{365}{100}=\frac{A}{86}$	$31390=100A$ $A=313.90$
52.2 is what percent of 870? Amount Rate Base	$\frac{r}{100}=\frac{52.2}{870}$	$870r=5220$ $r=6$ Rate=6%
Find 225% of 90. Rate Base Amount	$\frac{225}{100}=\frac{A}{90}$	$20250=100A$ $A=202.50$

TRY: Complete the following.

What is 9.5% of 700?		
350 is what percent of 200?		
Find the base if 130% of the base is 780.		
28.8 is what percent of 960?		
What is 225% of 48?		
Find 150% of 50.		

Unknown Amount, Rate, Base

TRY: You can also think of the rate proportion as: $\frac{r}{100} = \frac{part}{whole}$

Unknown Amount

In order to pass an exam, a student needs to answer 80% of the exam questions correctly. If the exam contains 24 questions, how many must the student answer correctly to pass? (Since the answer needs to be "whole" questions, be sure to round the answer UP.)

Unknown Rate

A chemist prepares a 400-ml. acid-water solution. If the solution contains 30 ml of acid, what percent of the solution is acid?

Unknown Base

A college finds that 35% of its science students take biology. If there are 252 biology students, how many science students are there altogether?

Tom has 6% of his salary deducted for a retirement plan. If that deduction is $168, what is his monthly salary?

A virus-scanning program is checking every file for viruses. It has completely scanned 30% of the files in 150 seconds. How long should it take to check all the files?

Commission, Sales Tax, Mark Up, Mark Down

When we multiply each side of the proportion $\frac{r}{100} = \frac{Amount}{Base}$ by the Base, we get: $Amount = \frac{r}{100} \bullet Base$

This equation is often useful when working with Commission and Sales Tax.

Commission and Sales Tax

CJ receives a 6% commission on everything sold. If CJ sold items for $1,240 during the summer, what would the amount of CJ's commission?

Set up: $Amount = \frac{r}{100} \bullet Base$ $\qquad$ $Amount = \frac{6}{100} \bullet 1240$

AG also worked during the summer for the same company, but at a different commission rate. If AG sold items for $500 and received $45 in commission, what was AG's rate of commission?

Set up: $\frac{r}{100} = \frac{Amount}{Base}$ $\qquad$ $\frac{r}{100} = \frac{45}{500}$

County sales tax is 5.5%. If the tax on a purchase was $90.75, what was the price of the purchase?

Set up: $\frac{r}{100} = \frac{Amount}{Base}$ $\qquad$ $\frac{5.5}{100} = \frac{90.75}{Base}$

TRY: Set up each problem and solve it.

JZ works on a 4% commission basis. JZ sold $45,000 in merchandise during 1 month. What was the amount of JZ's commission?

If the tax on a $130 purchase is $9.75, what is the sales tax in that state?

If CJ had received an 8% commission, how much would CJ need to sell to receive $1,240 in commission?

Mark Up and Mark Down

A store marks up items to make a 20% profit. If an item costs $6.80 from the supplier, what will be the amount of profit? What will be the Sale price? (Profit, increase, Mark Up are all similar terms.)

Set up for Profit: $Amount = \frac{r}{100} \bullet Base \quad Amount = \frac{20}{100} \bullet 6.80 \quad Amount = 1.36$

Set up for Sale Price: OriginalPrice + MarkUp = SalePrice

$$6.80 + 1.36 = 8.16$$

Shirts that normally sell for $25 are marked down 10%. What will be the amount of discount? What is the final sale price before taxes? (Discount, decrease, Mark Down are all similar terms.)

Set up for Discount: $Amount = \frac{r}{100} \bullet Base \quad Amount = \frac{10}{100} \bullet 25 \quad Amount = 2.50$

Set up for Sale Price: OriginalPrice – MarkDown = SalePrice

$$25 - 2.50 = 22.50$$

The furniture store increased the price of the newer chair by 12%. If the increase was $13.92, what was the original price before the increase?

Set up for Increase: $\frac{r}{100} = \frac{Amount}{Base} \quad \frac{12}{100} = \frac{13.92}{Base} \quad Base = 116$

TRY: Set up each problem and solve it.

If the 5.5% tax on a purchase was $90.75, what was the price of the purchase?

A new fax machine was recently purchased for an office for $464.40 including tax. If the tax rate is 8%, find the price of the fax machine before tax. CAREFUL – this one is different.

At the end of the semester, rental textbooks go on sale at 20% off. Find the sale price of a $110 book.

The price of a new compact car has increased $819 over the previous year. If this amounts to a 4.5% increase, what was the price of the car before the increase?

Simple and Compound Interest

Vocabulary

Principal	Base amount of money
Interest	Percent of the whole; amount charged on principal or earned on principal
Interest rate	Percentage rate charged or earned

$$Interest = \frac{r}{100} \bullet principal$$

If you borrow $3,500 for one year with an interest rate of 7 ½ %, what is the amount of interest you must pay?

Set up: $I = \frac{7.5}{100} \bullet 3500 \qquad I = 262.50$

If you will be charged interest of $3,600 in one year on a loan with a rate of 4.5%, how much is the loan?

Set up: $\frac{4.5}{100} = \frac{3600}{P} \qquad P = 80000$

If you will earn interest of $56.25 on a savings account for one year that had a constant balance of $4500, what was the interest rate?

Set up: $\frac{r}{100} = \frac{56.25}{4500} \qquad r = 1.25 \qquad R = 1.25\%$

TRY: Set up each problem and solve it.

A savings bank offers 5.25% on 1-year time deposits. If you place $3,000 in an account, how much will you have at the end of the year?

If you will be charged interest of $1,800 in one year on a loan with a rate of 4.5%, how much is the loan?

If you will earn interest of $56.25 on a savings account for one year that had a constant balance of $2500, what was the interest rate?

If $500 is compounded at 8% for 3 years, what will the final total be?
(Hint: find the interest for the first year. Add that to the $500. Find the interest for the 2nd year. Add that. Now find the interest for the 3rd year. Add that. Round your final answer to the nearest penny.)