Lesson 07: Inequality, Other Numbers, Review

Inequality Signs, Other Numbers

Inequality Signs

Inequality signs are used to indicate the relationship of one number or value to another.

5 is **less than** 8 written $5 < 8$

-4 is **greater than** -6 written $-4 > -6$

-2 is **less than or equal to** 3 written $-2 \leq 3$

25 is **greater than or equal to** 20+5 written $25 \geq (20+5)$

6 is **not equal to** 5 written $6 \neq 5$

TRY: Indicate if the following statements are true or false.

17	<	25	True or False	36	>	39	True or False
(5 + 3)	≤	(3+5)	True or False	(14 – 6)	≥	(13 – 2)	True or False

Use the < or > symbol to make each statement true.

(5 + 6 – 3)	_____	25	36	_____	(27 – 9 + 10)
17	_____	(25 – 9)	36	_____	(39 – 2)

Other Numbers

So far, three types of numbers, found on the number line have been defined:
Whole numbers, Natural numbers, and Integers. Three more types of numbers need to be defined.

DEFINITION: A **rational number** is any number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

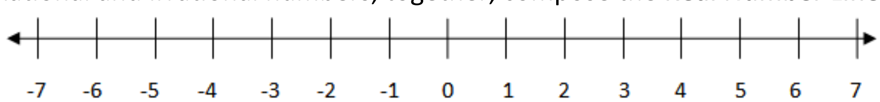
In other words, fractions such as $\frac{4}{9}$ and $\frac{13}{5}$ are rational numbers.

DEFINITION: **Irrational Numbers** are numbers that cannot be written as the quotient of two integers. When written in decimal form, an irrational number is a nonrepeating, nonterminating decimal.

The most common irrational number one uses is *pi*, π.

Definition: The Rational and Irrational numbers form the set of **Real Numbers**.

Rational and Irrational numbers, together, compose the **Real Number Line**:



The Number line extends from negative infinity to positive infinity. The infinity symbol is: ∞.

Properties of Real Numbers

Property Name	Property form	Example
If a and b are real numbers, then …		
Commutative property of Addition - The order of two numbers around the addition sign does not affect the sum.	$a+b=b+a$	$5+4=4+5$
Commutative property of Multiplication - The order of two numbers around the multiplication sign does not affect the product.	$a\cdot b=b\cdot a$	$5\cdot 4=4\cdot 5$

If a, b, and c are real numbers, then …		
Associative property of Addition - The way in which several whole numbers are grouped when they are added, does not affect the final sum.	$(a+b)+c=a+(b+c)$	$(3+4)+5=3+(4+5)$
Associative property of Multiplication - The way in which several whole numbers are grouped when they are multiplied, does not affect the final product.	$(ab)c=a(bc)$	$(3\cdot 4)\cdot 5=3\cdot(4\cdot 5)$

If a is a real number, then …		
Identity property of Addition - When 0 is added to any number, the sum is 'identical' to the original number. 0 is the identity element for addition; it is also called the additive identity.	$a+0=0+a=a$	$4+0=0+4=4$
Identity property of Multiplication - When any number is multiplied by 1, the product is 'identical' to the original number. 1 is the identity element for multiplication; it is also called the multiplicative identity.	$a\cdot 1=1\cdot a=a$	$3\cdot 1=1\cdot 3=3$

If a and b are real numbers, then …		
Inverse property of Addition - The sum of a number and its additive inverse is zero (the identity element for addition). The additive inverse of a is $-a$. The additive inverse of $-a$ is a.	$a+(-a)=-a+a=0$	$5+(-5)=-5+5=0$
Inverse property of Multiplication - Multiplying a number and its reciprocal (its multiplicative inverse) gives 1 (the identity element for multiplication). The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.	$b\cdot\frac{1}{b}=\frac{1}{b}\cdot b=1$ when $b\neq 0$	$3\cdot\frac{1}{3}=\frac{1}{3}\cdot 3=1$

If a, b, and c are real numbers, then …		
Distributive property of Multiplication over Addition - To distribute a factor over a sum of two numbers within parentheses, multiply the factor by each number inside the parentheses then add the products. - The Distributive property also works over Subtraction.	$a(b+c)=ab+ac$ $a(b-c)=ab-ac$ $(b+c)a=ba+ca$ $(b-c)a=ba-ca$	$7(5+3)=7\cdot 5+7\cdot 3$ $7(5-3)=7\cdot 5-7\cdot 3$ $(7+2)3=7\cdot 3+2\cdot 3$ $(7-2)3=7\cdot 3-2\cdot 3$

TRY: What is the identity element for addition?
What is the identity element for multiplication?

What is the additive inverse of -5? What is the reciprocal of $\frac{3}{5}$?

Sometimes one must use the distributive property before combining like terms.

$3(x^2+8)-2(x^2-5)$ Use the Distributive Property

$3x^2+24-2x^2+10$ Did you distribute the – with the -5 to get +10?
This is one of the most COMMON ERRORS! Be very careful.

x^2+34 Like terms combined.

TRY: Combine like terms in the following:

$-4(x+3y)+5(2x-y)$ $-4(x-3y)-5(-2x-y)$

Algebra: Solving Linear Equations in One Variable

To solve a linear equation:

1. Use the distributive property to remove any parentheses.
2. If multiple fractions are present, use the multiplication property of equality and multiply each term of the equation by the LCD. This will eliminate the denominators from the equation.
3. Combine any like terms.
4. Use the addition (or subtraction) property of equality to move all constants to one side of the equation and all terms with variables to the other side of the equation.
5. Use the multiplication or division property of equality to isolate the variable and reveal the solution.
6. Check your solution by replacing the variable in the original equation with your solution.

Example: Solve $x+7=-5$ for x.

$x+7=-5$	Use the subtraction property of equality to move the 7 to the right side.
$x+7-7=-5-7$	Simplify. $x+7-7=-5-7;\ x+0=-12;\ x=-12$
$x=-12$	Check the solution. $-12+7=-5$; $-5=-5$ is true. The solution set is: $\{-12\}$

If the resulting expression is an equation that is true for some values and not for others, such as $x=5$, the equation is called a **conditional equation** and has **only one solution**. The solution set is $\{5\}$.

Example: Solve $8=g-3$ for g.

$8=g-3$	Use the addition property of equality to move the -3 to the left side.
$8+3=g-3+3$	Simplify. $8+3=g-3+3;\ 11=g+0;\ 11=g$
$11=g$	Check the solution. $8=11-3;\ 8=8$ is true. The solution set is: $\{11\}$

Example: Solve $x+7=4+x+3$ for x.

$x+7=4+x+3$	Combine like terms on the right.
$x+7=x+7$	Use the subtraction property of equality to move the variable to the left side and 7 to the right. $x-x+7-7=x-x+7-7$
$0=0$	The statement is TRUE. Therefore the statement is an IDENTITY and the solution set is the set of all real numbers. The solution set is: $\{x \mid x \in REALS\}$

If the variable dropped out of the equation leaving a **true** statement, such as 7 = 7, the equation is called an **identity** and has **all real numbers** as its solution. The solution set is {all Reals}.

Example: Solve $x+7=x-5$ for x.

$x+7=x-5$	Use the subtraction property of equality to move the 7 to the right side.
$x+7-7=x-5-7$	Simplify. $x+7-7=x-5-7$; $x=x-12$; now move the x: $x-x=x-x-12$
$0=-12$	This statement is FALSE. Therefore there is NO solution. The result is the empty set: $\varnothing$

If the variable dropped out of the equation leaving a **false** statement, such as $8=-3$, the equation is called a **contradiction** and has **no solution**. The solution set is the **empty set**, $\varnothing$.

TRY: Solve $x+9=-7$ for x. Solve $16=x-4$ for x.

Example: Solve $5x=2x-12$ for x.

$5x=2x-12$ $5x-2x=2x-2x-12$	Use the subtraction property of equality to move the $2x$ to the left side to get the variable terms on the same side.
$3x=-12$ $\frac{3x}{3}=\frac{-12}{3}$	Combine like terms. Use the division property of equality to divide both sides by 3 to isolate the variable.
$x=-4$	Simplify. Check the solution. $5(-4)=2(-4)-12$; $-20=-8-12$; $-20=-20$ is true. The solution is {-4}

Example: Solve $\frac{2}{5}x-7=9$ for x.

$\frac{2}{5}x-7=9$ $\frac{2}{5}x-7+7=9+7$	Use the addition property of equality to move the -7 to the right side to isolate the variable term.
$\frac{2}{5}x=16$ $\frac{5}{2}\bullet\frac{2}{5}x=\frac{5}{2}\bullet 16$	Combine like terms. Use the multiplication property of equality to multiply both sides by the reciprocal to isolate the variable. Reduce.
$1\bullet x=5\bullet 8$ $x=40$	Simplify. Check the solution. $\frac{2}{5}(40)-7=9$; $2\bullet 8-7=9$; $16-7=9$; $9=9$ is true. The solution is {40}

WARNING – UNDERSTANDING how to solve for the variable is CRITICAL to this course. Be sure you understand these concepts and can successfully solve for the variable.

TRY: Solve each of the following for the variable.

$3x - 5 = 7$

$5 - 6x = -19$

$\frac{2}{5}x + 7 = 17$

$8x - 6 = 4x + 18$

$-3(2x + 4) = -10x$

$4(-x - 2) = -2(2x + 7) - 5$