Algebra: Solving Linear Equations in One Variable

To solve a linear equation:

1. Use the distributive property to remove any parentheses.

2. If multiple fractions are present, use the multiplication property of equality and multiply each term of the equation by the LCD. This will eliminate the denominators from the equation.

3. Combine any like terms.

4. Use the addition (or subtraction) property of equality to move all constants to one side of the equation and all terms with variables to the other side of the equation.

5. Use the multiplication or division property of equality to isolate the variable and reveal the solution.

6. Check your solution by replacing the variable in the original equation with your solution.

Example: Solve $x+7=-5$ for x.

$x+7=-5$	Use the subtraction property of equality to move the 7 to the right side.
$x+7-7=-5-7$	Simplify. $x+7-7=-5-7$; $x+0=-12$; $x=-12$
$x=-12$	Check the solution. $-12+7=-5$; $-5=-5$ is true. The solution set is: $\{-12\}$

> If the resulting expression is an equation that is true for some values and not for others, such as $x=5$, the equation is called a **conditional equation** and has **only one solution**. The solution set is $\{5\}$.

Example: Solve $8=g-3$ for g.

$8=g-3$	Use the addition property of equality to move the -3 to the left side.
$8+3=g-3+3$	Simplify. $8+3=g-3+3$; $11=g+0$; $11=g$
$11=g$	Check the solution. $8=11-3$; $8=8$ is true. The solution set is: $\{11\}$

Example: Solve $x+7=4+x+3$ for x.

$x+7=4+x+3$	Combine like terms on the right.
$x+7=x+7$	Use the subtraction property of equality to move the variable to the left side and 7 to the right. $x-x+7-7=x-x+7-7$
$0=0$	The statement is TRUE. Therefore the statement is an IDENTITY and the solution set is the set of all real numbers. The solution set is: $\{x \mid x \in REALS\}$

> If the variable dropped out of the equation leaving a **true** statement, such as 7 = 7, the equation is called an **identity** and has **all real numbers** as its solution. The solution set is {all Reals}.

Example: Solve $x+7 = x-5$ for x.

$x+7 = x-5$	Use the subtraction property of equality to move the 7 to the right side.
$x+7-7 = x-5-7$	Simplify. $x+7-7 = x-5-7$; $x = x-12$; now move the x: $x-x = x-x-12$
$0 = -12$	This statement is FALSE. Therefore there is NO solution. The result is the empty set: $\varnothing$

If the variable dropped out of the equation leaving a **false** statement, such as $8 = -3$, the equation is called a **contradiction** and has **no solution**. The solution set is the **empty set**, $\varnothing$.

TRY: Solve $x+9 = -7$ for x.   Solve $16 = x-4$ for x.

Example: Solve $5x = 2x-12$ for x.

$5x = 2x-12$ $5x-2x = 2x-2x-12$	Use the subtraction property of equality to move the $2x$ to the left side to get the variable terms on the same side.
$3x = -12$ $\dfrac{3x}{3} = \dfrac{-12}{3}$	Combine like terms. Use the division property of equality to divide both sides by 3 to isolate the variable.
$x = -4$	Simplify. Check the solution. $5(-4) = 2(-4)-12$; $-20 = -8-12$; $-20 = -20$ is true. The solution is {-4}

Example: Solve $\dfrac{2}{5}x-7 = 9$ for x.

$\dfrac{2}{5}x-7 = 9$ $\dfrac{2}{5}x-7+7 = 9+7$	Use the addition property of equality to move the -7 to the right side to isolate the variable term.
$\dfrac{2}{5}x = 16$ $\dfrac{5}{2} \bullet \dfrac{2}{5}x = \dfrac{5}{2} \bullet 16$	Combine like terms. Use the multiplication property of equality to multiply both sides by the reciprocal to isolate the variable. Reduce.
$1 \bullet x = 5 \bullet 8$ $x = 40$	Simplify. Check the solution. $\dfrac{2}{5}(40)-7 = 9$; $2 \bullet 8-7 = 9$; $16-7 = 9$; $9 = 9$ is true. The solution is {40}

WARNING – UNDERSTANDING how to solve for the variable is CRITICAL to this course. Be sure you understand these concepts and can successfully solve for the variable.

TRY: Solve each of the following for the variable.

$3x - 5 = 7$ $5 - 6x = -19$

$\dfrac{2}{5}x + 7 = 17$ $8x - 6 = 4x + 18$

$-3(2x + 4) = -10x$ $4(-x - 2) = -2(2x + 7) - 5$