Lesson 05: Decimals

Decimals: Introduction, Place Value, and Rounding

Place Value:

Hundreds	Tens	Ones	AND	Tenths	Hundredths	Thousandths	Ten thousandths	Hundred thousandths
7	4	9	.	2	1	8	9	6

Seven hundred forty-nine AND twenty-one thousand, eight hundred ninety-six hundred thousandths

Reading a decimal: 749.21896

1. Read the digits to the left of the decimal point as a whole number. *Seven hundred forty-nine*
2. Red the decimal point as *AND*
3. Read the digits to the right of the decimal point as a whole number followed by the place value of the rightmost digit. *twenty-one thousand, eight hundred ninety-six hundred thousandths*

TRY: Write the following in words.

3.025

12.009

Vocabulary

Decimal Fraction	A fraction whose denominator is a power of 10

Hundreds	Tens	Ones	AND	Tenths	Hundredths	Thousandths	Ten thousandths	Hundred thousandths
10^2	10^1	10^0	.	$\frac{1}{10^1}$	$\frac{1}{10^2}$	$\frac{1}{10^3}$	$\frac{1}{10^4}$	$\frac{1}{10^5}$
100	10	1		.1	.01	.001	.0001	.00001
7	4	9	.	2	1	8	9	6

Examples:

13.07 is read thirteen and seven hundredths. As a mixed number it is: $13\frac{7}{100}$

245.125 is read two hundred forty-five and one hundred twenty-five thousands.

As a mixed number it is: $245\frac{125}{1000}$ which simplifies to: $245\frac{1}{8}$

TRY: Complete the following table (do not simplify).

Mixed Number	Decimal	Mixed Number	Decimal
	35.4	$3\frac{23}{10000}$	
$7\frac{3}{10}$			12.009
	3.025	$13\frac{7}{100}$	

Which is larger?

To compare two decimals, line up the decimal points. If one has fewer digits to the right of the decimal point than the other, add zeros as needed. Compare the two values.

Example:

.228 > .215 .3266 ? .327 (change to .3270 and compare)

.3266 < .3270

TRY: Arrange the following in order from smallest to largest.

.61 .061 $\frac{6}{100}$ .0059 $\frac{6}{10}$

Process of Rounding:

Round .52634 to the nearest .001 Rounded answer: .526	3. Look at the digit to the right of the .001's place. (Look at the 3.) 4. Since the digit is less than 5, discard that digit and all others to the right. (i.e., discard the 3 and the 4)

Round 17.648 to the nearest .01. Rounded answer: 17.65	1. Look at the digit to the right of the .01's place. (Look at the 8.) 2. Since the digit is 5 or greater, increase the value of the .01's place by one and discard all other digits to the right. (i.e., increase the 4 to a 5 and discard the 8)

TRY:

Round each to the …	Nearest tenths	Nearest hundredths	Nearest thousandths
8.3238			
14.9794			
5.0723			

Careful! If asked to round to a specific place, be sure to have a digit remain in that place – even if the digit is 0.

Decimals: Addition and Subtraction

Adding Decimals:
When adding decimals, ALIGN the decimal points! Add just as you would add whole numbers.
The decimal point of the sum should align with the decimal point of the addends.

Example: (fill in the missing 0 if you desire)

$$\begin{array}{r} 493.7 \\ 39 \\ +\ 14.4 \\ \hline 547.1 \end{array}$$

TRY:

$$\begin{array}{r} 73.02 \\ 3.3 \\ +\ 76.8 \\ \hline \end{array}$$

TRY:

Add 32.1, 3.67, 5, and 6.32.

Subtracting Decimals:
When subtracting decimals, ALIGN the decimal points! Add zeros to the right of any number to right align all the digits. Subtract as you would with whole numbers. The decimal point of the difference should align with the decimal point of the number being subtracted.

Example:

$$\begin{array}{r} 39.0 \\ -\ 14.4 \\ \hline 24.6 \end{array}$$

TRY:

$$\begin{array}{r} 53.38 \\ -\ 6.884 \\ \hline \end{array}$$

TRY:

Subtract 3.67 from 32.1

When adding or subtracting negative decimals, follow the rules for working with integers.

Example:

Subtract 5.29 from -7.13 Set up: -7.13 - 5.29 rewrite: - 7.13 + - 5.29 (now add these) = -12.42

Subtract -3.25 from -6.4 Set up: -6.40 - (-3.25) rewrite: -6.40 + 3.25 (take the difference) = -3.15

TRY:

Subtract 7.89 from -8.14

Subtract -6.3 from -9.36

Decimals: Multiplication

Multiplying Decimals:

When multiplying decimals, multiply the decimals as if they were whole numbers. Count the number of decimal places in each of the numbers being multiplied. Place the decimal point in the product so that the number of decimal places in the product is the sum of the number of decimal places in the numbers being multiplied.

Example:

$$\begin{array}{r} 39.3 \\ \times \quad 4.45 \\ \hline 1965 \\ 1572 \\ 1572\_ \\ \hline 174.885 \end{array}$$

A total of 3 decimal places are needed in this answer.

TRY:

$$\begin{array}{r} 53.38 \\ \times \quad 6.884 \\ \hline \end{array}$$

Multiplying by Powers of 10:

When multiplying decimals by powers of 10, move the decimal point in the original number to the right the same number of places as zeros in the powers-of-10 number.

Examples: $36.493 \times 100 = 3649.3$ (decimal moved 2 places to the right)
$36.493 \times 10^2 = 3649.3$

$.3942 \times 1000 = 394.2$ (decimal moved 3 places to the right)
$.3942 \times 10^3 = 394.2$

$.057 \times 100000 = 5700$ (decimal moved 5 places to the right and zeros added)
$.057 \times 10^5 = 5700$

Notice how the number of places one moves the decimal to the right
is the same as the exponent of the power-of-10 term used for multiplication.

TRY:

23.5804×100 $\qquad$ 4.59×10^4 $\qquad$ $.0025 \times 10^7$

Decimals: Division

Dividing a Decimal by a Whole Number:

When dividing a decimal by a whole number, place the decimal point in the quotient directly above the decimal point in the dividend.

$$\begin{array}{r} 5.48 \\ 5\overline{)27.4} \\ \underline{25} \\ 24 \\ \underline{20} \\ 40 \\ \underline{40} \\ 0 \end{array}$$ Continue dividing, by adding zeros, until the answer: 1 – terminates or 2 – reaches the place value you want to use for rounding (such as hundredths for the nearest tenth)	TRY: $8\overline{)47.8}$

Dividing Decimals by a Decimal:

When dividing a decimal by another decimal, move the decimal in the divisor to the right, making the divisor a whole number. Move the decimal point in the dividend to the right the same number of places, adding zeros as necessary. Divide as you would with whole numbers, placing the decimal point in the quotient directly above the decimal point of the new dividend.

Divide : 0.91728 ÷ .42 $.42\overline{)0.91728}$ $$\begin{array}{r} 2.184 \\ 42.\overline{)091.728} \\ \underline{84} \\ 77 \\ \underline{42} \\ 352 \\ \underline{336} \\ 168 \\ \underline{168} \\ 0 \end{array}$$ moved decimal right 2 places	TRY: $1.2\overline{)7.848}$

Dividing by Powers of 10:

When dividing decimals by powers of 10, move the decimal point in the original number to the left the same number of places as zeros in the powers-of-10 number. Add zeros as necessary.

Examples:

3649.3 ÷ 100 = 36.493 (decimal moved 2 places to the left)
$3649.3 \div 10^2 = 36.493$

394.2 ÷ 1000 = .3942 (decimal moved 3 places to the left)
$394.2 \div 10^3 = .3942$

5.7 ÷ 100000 = .000057 (decimal moved 5 places to the left and zeros added)
$5.7 \div 10^5 = .000057$

Notice how the number of places one moves the decimal to the left
is the same as the exponent of the power-of-10 term used for division.

TRY:

$23.5804 \div 100$ $4.59 \div 10^4$ $.0025 \div 10^7$

The rules for signs (the quotient of two numbers with the same sign is positive; the quotient of two numbers with opposite signs is negative) and the rules governing Order of Operations apply for decimals.

TRY:

$-15.75 \div 2.5$ $-11.02 \div -2.9$

$$3.4 - 2(4.6 - 6.4)^2 - 5.3 \bullet 3.2$$

Decimals and Fractions

Converting a fraction to a decimal:

To convert a fraction to a decimal, divide the numerator of the fraction by its denominator.

Vocabulary

Decimal Equivalent of a fraction	The decimal formed by dividing the numerator of the fraction by its denominator.
Terminating Decimal	A decimal equivalent that stops when a 0 remainder is reached
Repeating Decimal	A decimal equivalent that ends with a repeating sequence of digits
Bar Notation	The bar placed over the repeating digits of a repeating decimal

TRY:

Complete this chart of decimal equivalents by dividing the numerator of each fraction by its denominator. If the decimal equivalent is a repeating decimal, be sure to use bar notation.

$\frac{1}{2}=0.5$	$\frac{1}{3}=0.\bar{3}$	$\frac{1}{4}=0.25$	$\frac{1}{5}=0.2$	$\frac{1}{6}=0.1\bar{6}$	$\frac{1}{8}=0.125$
	$\frac{2}{3}=$	$\frac{3}{4}=$	$\frac{2}{5}=$	$\frac{5}{6}=0.8\bar{3}$	$\frac{3}{8}=$
			$\frac{3}{5}=0.6$		$\frac{5}{8}=$
			$\frac{4}{5}=$		$\frac{7}{8}=0.875$
$\frac{1}{7}=0.\overline{142857}$		$\frac{1}{9}=0.\bar{1}$	$\frac{1}{11}=0.\overline{09}$	$\frac{1}{12}=0.08\bar{3}$	$\frac{1}{13}=0.\overline{076923}$
$\frac{2}{7}=0.\overline{285714}$		$\frac{2}{9}=$	$\frac{2}{11}=$	$\frac{5}{12}=$	$\frac{2}{13}=$
$\frac{3}{7}=$		$\frac{5}{9}=$	$\frac{9}{11}=$	$\frac{11}{12}=0.91\bar{6}$	$\frac{11}{13}=$

To convert a mixed number to a decimal equivalent, find the decimal equivalent of the fractional part and add the whole number part.

Example:

$$5\frac{2}{3}=5.\bar{6}$$

TRY:

$$24\frac{5}{8}=$$

Converting a terminating decimal to a fraction:

If the decimal value is less than 1 –

Write the digits of the decimal without the decimal point in the numerator of the fraction.
Write the place value of the rightmost digit in the denominator of the fraction.
Simplify if possible.

Example: TRY:

$$.35 = \frac{35}{100} = \frac{7}{20} \qquad .031 =$$

If the decimal value is greater than or equal to 1 (i.e., it has a whole number portion) -

Write the digits to the right of the decimal as a simplified proper fraction and then form a mixed number with the whole number portion.

Example: TRY:

$$6.35 = 6\frac{35}{100} = 6\frac{7}{20} \qquad 4.08 =$$

Comparing fractions and decimals:

Find the decimal equivalent of the fraction and compare the decimals.

Examples:

Which is larger $\frac{3}{8}$ or 0.38? $\frac{3}{8}$ converts to 0.375 0.38 can be expanded to 0.380

0.375 < 0.380 So $\frac{3}{8}$ < 0.38

Which is larger $\frac{7}{9}$ or 0.76? $\frac{7}{9}$ converts to $0.\overline{7}$ which, when expanded is: $0.77\overline{7}$

$0.77\overline{7}$ > 0.76 So $\frac{7}{9}$ > 0.76

TRY:

Which is larger? $\frac{2}{15}$ or 0.15

Decimals in Equations

When working with decimals in equations, carefully use the Addition, Subtraction, Multiplication, and Division Properties Equality to isolate the variable. (i.e., whatever you do to one side of the equation, do the same to the other.)

Example: Solve $x+1.7=-5$ for x.

$x+1.7=-5$	Use the subtraction property of equality to subtract 1.7 from both sides.
$x+1.7-1.7=-5-1.7$	Simplify. $x+1.7-1.7=-5+-1.7;\ x+0=-6.7;\ x=-6.7$
$x=-6.7$	Check the solution. $-6.7+1.7=-5\ ;\ -5=-5$is true.

Example: Solve $4.2x-8=3.4x+7$ for x.

$4.2x-8=3.4x+7$	Use the addition or subtraction property to combine like variables on one side and the constants on the other.
$4.2x-8+8=3.4x+7+8$ $4.2x=3.4x+15$ $4.2x-3.4x=3.4x-3.4x+15$ $0.8x=15$ $\frac{0.8}{0.8}x=\frac{15}{0.8}$	Use the addition property to add 8 to both sides. Combine like terms. Use the subtraction property to subtract $3.4x$ from both sides. Combine like terms. Use the division property to divide both sides by .08
$x=18.75$	Check the solution. $4.2(18.75)-8=3.4(18.75)+7;\ \ 70.75=70.75$ is true.

Sometimes, it is easier to work without decimals. To change a problem with decimals to one without, multiply EVERY TERM in the equation by the same power of 10.

For example, since the above problem involved tenths, one would multiply every term by 10.
The problem would then becomes: $42x-80=34x+70$ ➔ $8x=150$➔ $x=18.75$
The answer is still the same.

If the problem involved hundredths, one would multiply every problem by 100.
For example, one would multiply every term in $2.3x-4.25=3.3x+2.15$ by 100.
This changes the problem to: $230x-425=330x+215$

TRY: Solve this equation both ways – keep the decimals (left) and solve the multiplied problem (right).
$2.3x-4.25=3.3x+2.15$ $230x-425=330x+215$

TRY: Solve for the value of x.

$2.4x = 2.16$

$2.7x + 5.4x = -16.2$

$3.2x - 8.4 = 1.2x - 0.4$

$0.8x - 2.1 = 0.3x - 0.1$

$0.05(x + 8) - 0.01x = 0.6$

$1.2(2x + 5) - 2.1 = 1.5(x + 5)$

Square Roots

Vocabulary

Square Root	A factor of a number when multiplied by itself yields the number Ex: the square root of 9 is 3 since 3 · 3 = 9
Squared	Multiplying a number times itself. Ex: 3^2 = 3 · 3 = 9
Radical sign, $\sqrt{\ }$	The symbol used to indicate square root. Ex: $\sqrt{9}$
Perfect Square	A whole number that is the product of a factor squared. Ex: 9 is a perfect square since 3^2 = 3 · 3 = 9.

To find the square root of a number, ask what number squared equals that number.

What squared equals 49?

What number multiplied by itself equals 49?

$7 \cdot 7 = (7)^2 = 49$ so $\sqrt{49} = 7$

The $\sqrt{\ }$ symbol represents ONLY the positive square root $\sqrt{49} = 7$ even though $(-7)^2 = 49$.

TRY:

$\sqrt{25}$ $\sqrt{121}$ $\sqrt{225}$

$-\sqrt{64}$ $\sqrt{64}$ $-\sqrt{25}$

Sometimes, the result isn't an integer. What is $\sqrt{29}$? Since $\sqrt{29}$ is between $\sqrt{25}$ and $\sqrt{36}$, the answer must be between 5 and 6. Since 29 is closer to 25, the answer will be closer to 5 than to 6.

TRY: What is the approximate answer for $\sqrt{7}$?

Root Chart $\sqrt{a} = b$

b ➔	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\sqrt{a}$	4	9	16	25	36	49	64	81	100	121	144	169	196	225

Decimals in Applications

TRY: Solve the following application problems.

1. The last time I went on a trip, I started and ended the trip with a full tank of gas. I filled my tank three times with 8.3, 8.65, and 8.7 gallons of fuel.
   a. What was the total amount of fuel used?
   b. If I paid $24, $24.96, and $25.66 for fuel and $0.40 in tolls, what was the total amount paid for fuel and toll?
   c. If I averaged 31.49 miles per gallon, what is the approximate number of miles I drove (to the nearest mile)?

2. During the trip, I found a great sale. A camera that originally cost $236.79 was marked down to $184.99. How much did I save on the camera?

3. Our sailboat, EZ Livin', consumes 0.75 gallons of diesel per hour when the engine is used to power the boat. Last weekend, the winds were light, so we had to lower the sails and use the engine $3\frac{1}{4}$ hours. How much diesel fuel was consumed during that time?

4. CK always writes down a deposit or check amount in the company checkbook, but doesn't maintain a current balance. If the company checking account showed a balance of $1,589.47 and the following entries were made, what is the current balance? At any time, was the account overdrawn (the balance went below $0)?

Beginning balance:	$1,589.47
Deposit	1,600.93
Check	2,853.85
Check	928.51
Deposit	3,439.42

Statistics: Mean, Median, Mode

Vocabulary

Average	The number that is typical of a group of numbers
Mean	A form of average. The mean is found by adding all the numbers in a group together and dividing that sum by the number of items in the group. The mean of 4, 7, 9, 8, 5, 9 is 7
Median	A form of average. The median is the middle number in the list of all the numbers in the group when placed in order from smallest to largest. If two numbers are in the middle, the median is the mean of the two middle numbers. The median of 4, 7, 9, 8, 5, 9 is 7.5
Mode	A form of average. The mode is number or item that appears Most Often. The mode of 4, 7, 9, 8, 5, 9 is 9.

TRY:

Mean – the average (add up the numbers in the group and divide by the number of items in the group)

1. Find the mean of: 12, 16, 15, 40, 12

2. Find the mean of: 3, 4, 6, 5, 2, 4, 9, 7

Median – the middle value (order the numbers; find the exact middle value or ½ way between the middle 2)

3. Find the median of: 12, 16, 15, 40, 12

4. Find the median of: 3, 4, 6, 5, 2, 4, 9, 7

Mode – the most often (find the number or item that occurs the most frequently in the list)

5. Find the mode of: 12, 16, 15, 40, 12

6. Find the mode of: 3, 4, 6, 5, 2, 4, 9, 7

Mode can also be used with something other than numbers

7. Find the mode of the fruit in the bowl: apple, banana, pear, banana, pear, apple, pear

Ratios

Vocabulary

Ratio	A means of comparing two numbers or quantities that have the same units. A ratio can be expressed in a variety of ways: 3 to 5 3:5 $\frac{3}{5}$

Example:

Length to Width 18" to 10" 18" : 10" $\frac{18"}{10"}$ which can be reduced to $\frac{9}{5}$

We can interpret this to mean, for every 9 inches of length, the object has 5 inches of width.

A ratio is correctly written when both numbers or quantities have the same units.

1.5 feet to 10 inches is not a ratio until it is changed to 18 inches to 10 inches.

Example:

CJ ran $4\frac{1}{3}$ miles for every $1\frac{1}{5}$ miles MG walked. Express this ratio as a ratio of whole numbers.

First, write the ratio as a complex fraction. $\frac{4\frac{1}{3}}{1\frac{1}{5}}$. Then, evaluate it: $\frac{\frac{13}{3}}{\frac{6}{5}} = \frac{13}{3} \bullet \frac{5}{6} = \frac{65}{9}$

CJ runs 65 miles for every 9 miles that MG walks.

TRY: Express each of the following as ratios of whole numbers. Be careful to have **similar units**.

TL takes $1\frac{1}{2}$ hours to complete unit 5, while JT takes 45 minutes. Find the ratio of the two times.

Conveyor A moves 2.4 feet of board in the same time that Conveyor B moves 5.76 feet of board. Find the ratio of the two conveyors.

Rates

Vocabulary

Rate	A comparison of two numbers or quantities with different units EX: $\frac{3\,ft}{5\,\text{sec}}$ $\frac{\$10}{3\,yr}$
Unit rate	A rate with the numerical portion separated from the unit's portion indicating a certain number of numerator units per one denominator unit. EX: $15\frac{mi}{gal}$ is read 15 miles per 1 gallon
Unit price	Price for a common unit. EX: $25\frac{cents}{quart}$ 25 cents per quart

TRY: Find the Unit rate for each of the following.

JT scored 650 points in 25 games. KS scored 350 in 15 games. Who had the higher points/game rate?

With the Forrester last summer, I was able to travel 258 miles on 10.75 gallons.
What was the miles per gallon rate?

TRY: Find the Unit Price for each of the following.

Last week, I was able to purchase 12 cans of Coke for $3.60. What was the unit price?

In ordering CDs, I was able to get 8 CDs for $119.92. What was the price paid per unit?

SK paid $1.29 for 22 oz bottle of gel. GT paid $0.95 for a 14 oz bottle of gel. Who paid the least per oz?