Lesson 04: Factors and Fractions

Factors, Prime Numbers, and GCF

Vocabulary

Factor	Each natural number used to form a product. A factor of a natural number is another natural number that will divide exactly into that number with 0 as the remainder. Since 6 · 4 = 24, then 6 and 4 are factors of 24. 24 ÷ 6 = 4, r 0 and 24 ÷ 4 = 6, r 0

Every number has at least two factors (or divisors) – the number itself and the number 1.
Since 7 · 1 = 7, then 7 and 1 are both considered to be factors of 7.

Vocabulary

Prime	A natural number, other than 1, whose only factors are the number 1 and itself.
Composite	A natural number greater than 1, that is not prime. (i.e. it has factors other than 1 and itself).

The number 1 is considered to be neither prime nor composite.

TRY:

Finish the list of the first ten prime numbers: 2 3 ___ ___ ___ ___ ___ ___ ___ 29

When working with division and other areas of mathematics, it is often helpful to write a number as a product of its prime factors. This is called finding the **prime factorization** of the number or **factoring** a number.

To find all the factors of a given number, one uses a **factor tree**.

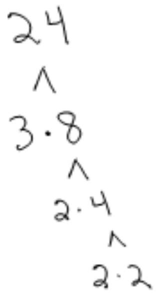
Example:

Write 24 as the product of its prime factors.

Two factors of 24 are 6 and 4.
Two factors of 6 are 3 and 2.
Two factors of 4 are 2 and 2.

It doesn't matter which factors one uses to start the factor tree. The results will be the same.

The prime factorization of 24 is
3 · 2 · 2 · 2



One continues to build the factor tree until the number at the end of each 'branch' is a prime number.
What about the prime factorization of 48?

The prime factorization of 48 = 2 · 3 · 2 · 2 · 2
Since multiplication is commutative, the order of the factors doesn't matter. We can rearrange the order so all the like factors are together. Then, to condense the result, we can use exponents. Since the 2 appears 4 times, the result can be written as: $48 = 2^4 \cdot 3$.

TRY: What is the prime factorization of 40?

Vocabulary

Common Factor or Divisor	A natural number that is a factor of two other numbers

What are the common factors of 18 and 24?
Staring with 1, list all the factors of 18. Then, list all the factors of 24.

18:	1	2	3		6		9		18		
24:	1	2	3	4	6	8		12		24	
36:	1	2	3	4	6		9	12	18		36
Common:	1	2	3		6						

The common factors of 18, 24, and 36 are: 1, 2, 3, 6

TRY: What are the common factors of 30 and 45?

Vocabulary

Greatest Common Factor (GCF)	The largest factor in common that will divide each of the given numbers exactly.

The common factors of 18, 24, and 36 are: 1, 2, 3, 6
So the GCF of 18, 24, and 36 is 6.

Finding all the factors of three numbers and then finding the greatest common factor takes a lot of time, especially if the numbers are large. There is another way.

Write the prime factorization of each number. (Use factor trees if you wish.)

18 = 2·9 = 2·3·3
24 = 2·12 = 2·2·6 = 2·2·2·3
36 = 6·6 = 2·3·2·3 = 2·2·3·3

Remember, it doesn't matter how you start the process, the result will be the same.

Line up the common factors found. It doesn't matter the order. Just be sure to line up like factors.

18:	2	3	3		
24:	2	3		2	2
36:	2	3	3	2	
common?	**2**	**3**			

The **GREATEST COMMON FACTOR** will be the product of all the common prime factors. If there are no factors in common, the GCF is 1.
The **GCF** of 18, 24, and 36 is 2·3 or **6**.

TRY: Find the GCF of 15 and 20. Find the GCF of 12, 24, and 48.

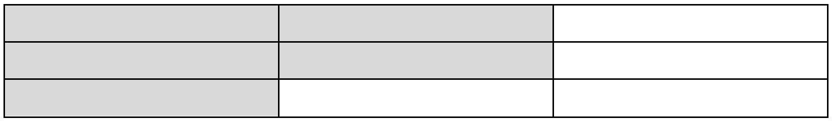
In algebra, it is often helpful to find factors with a given sum or difference. Complete the following.

Find 2 factors of	Whose sum is	**ANSWER**
10	7	5 and 2
28	11	
81	18	
36	13	
24	14	
Find 2 factors of	Whose difference is	**ANSWER**
10	9	10 and 1
18	7	
15	2	
12	1	

Fractions: An Introduction

Vocabulary

Fraction	A part of a whole quantity.



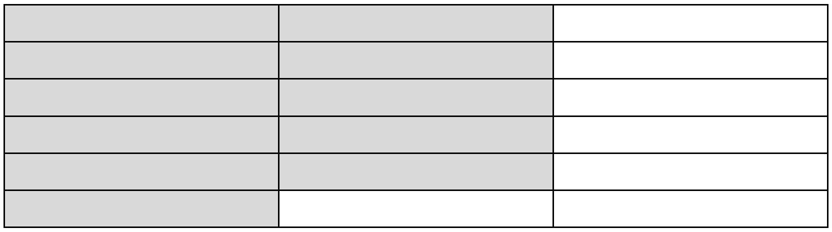
The shaded area represents 5 parts of the total 9 parts. This is represented by the fraction: $\frac{5}{9}$

A fraction is commonly written in the form: $\frac{a}{b}$ where $b \neq 0$.

A fraction can also be thought of as representing division: $a \div b$

TRY:

What fraction represents the shaded part?



Vocabulary

Numerator	The number, a, on top of the fraction.
Denominator	The number, b, on the bottom of the fraction.
Proper Fraction	A fraction where the numerator is less than the denominator. It names a number less than 1. Examples: $\frac{5}{9}$ $\frac{2}{3}$ $\frac{3}{8}$
Improper Fraction	A fraction where the numerator is greater than or equal to the denominator. It names a number greater than or equal to 1. Examples: $\frac{9}{5}$ $\frac{3}{2}$ $\frac{8}{3}$

TRY:

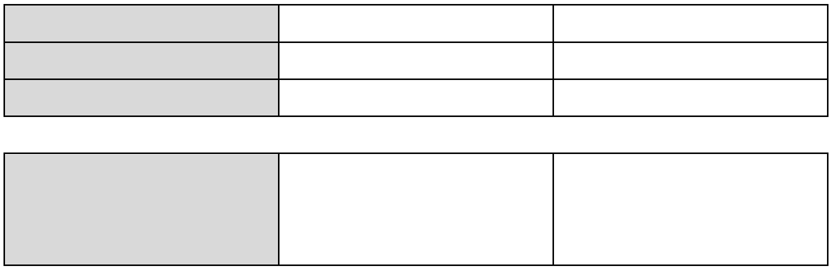
Identify each number as a proper fraction or an improper fraction.

$$\frac{9}{17} \qquad \frac{3}{29} \qquad \frac{80}{3} \qquad \frac{9}{9} \qquad \frac{3}{7} \qquad \frac{8}{13}$$

Fractions: Equivalent Fractions

Vocabulary

Equivalent Fractions	Two fractions representing the same portion.



$\frac{3}{9}$ represents the same portion as $\frac{1}{3}$ and are said to be equivalent fractions.

Vocabulary

Cross Product	Given two fractions: $\frac{a}{b} = \frac{c}{d}$, $a \bullet d$ and $b \bullet c$ are cross products.

Test for Equivalent Fractions: Two fractions are equivalent if the cross products are equal.

Are $\frac{3}{9}$ and $\frac{1}{3}$ equivalent fractions? Does $3 \cdot 3 = 9 \cdot 1$? Yes. They are equivalent fractions.

TRY:

Are $\frac{2}{7}$ and $\frac{6}{21}$ equivalent fractions?

Are $\frac{2}{5}$ and $\frac{9}{15}$ equivalent fractions?

Fundamental Principles of Fractions

$\frac{a}{b} = \frac{a \div c}{b \div c}$ for the fraction $\frac{a}{b}$ and any nonzero number c. As long as we divide both the numerator and the denominator by the same number, the result is an equivalent fraction.
$\frac{a}{b} = \frac{a \bullet c}{b \bullet c}$ for the fraction $\frac{a}{b}$ and any nonzero number c. As long as we multiply both the numerator and the denominator by the same number, the result is an equivalent fraction.

Vocabulary

Lowest terms or Simplest form	When the numerator and the denominator have no common factors other than the number 1, the fraction is said to be in lowest terms.

Reducing Fractions

To change, or reduce, a fraction into lowest terms, one writes the numerator and denominator as the product of their primes and then divides the numerator and the denominator by the same common factor.

$$\frac{8}{12}=\frac{2*\cancel{2}*\cancel{2}}{3*\cancel{2}*\cancel{2}}=\frac{2}{3}$$

$\frac{12}{8}$ is an *improper* fraction. It may be written as an improper fraction in lowest terms as $\frac{3}{2}$ or as a mixed number as $1\frac{1}{2}$. In either case, the proper form is in lowest terms. $\frac{3}{1}$ is an improper fraction. The correct form is just 3.

TRY: Reduce the following: $\frac{25}{5}=$ $\frac{49}{63}=$ $\frac{-36}{-24}=$

Making equivalent fractions

To make an equivalent fraction with a different denominator, one multiplies the numerator and the denominator of the original fraction by the same factor.

$\frac{3}{7}=\frac{?}{28}$ To form the denominator 28, the 7 was multiplied by 4.

$\frac{3}{7}=\frac{3\cdot4}{7\cdot4}=\frac{12}{28}$ To create the equivalent fraction, multiply the numerator by 4 as well.

TRY: Form equivalent fractions

$\frac{5}{9}=\frac{?}{36}$ $\frac{2}{11}=\frac{?}{66}$ $\frac{-3}{13}=\frac{?}{39}$

Vocabulary

Standard form	The standard form of a negative fraction is with the negative sign in the numerator or in front of the entire fraction. When used for evaluating an expression, the negative sign is considered with the numerator. $\frac{2}{-5}$ in standard form is $\frac{-2}{5}$ or $-\frac{2}{5}$

Divisibility Rules can help one quickly spot common factors.

A number is divisible by	**Example**
…2 if it is an even number.	7394 is divisible by 2 because it ends in 4, making it an even number.
…3 if the sum of its digits is divisible by 3.	837 -- Add the digits: 8 + 3 + 7 = 18. Since 18 is divisible by 3, the number 837 is divisible by 3.
…4 if its last two digits form a number that is divisible by 4.	5932 -- The last two digits form the number 32. Since 32 is divisible by 4, the number 5932 is divisible by 4.
…5 if the number ends in 0 or 5.	645 is divisible by 5 since it ends in 5.
…6 if it divisible by 2 and 3.	1248 – The number is divisible by 2 since it is an even number. The number is divisible by 3 since the sum of its digits is divisible by 3: 1 + 2 + 4 + 8 = 15. Therefore, the number 1248 is divisible by 6.
…9 if the sum of its digits is divisible by 9.	837 -- Add its digits: 8 + 3 + 7 = 18. Since 18 is divisible by 9, the number 837 is divisible by 9.
…10 if it ends in a zero.	890 is divisible by 10 because it ends in 0.

Determine if each of the following numbers is divisible by 2, 3, 4, 5, 6, 9, and/or 10.
(Mark the column with an X.)

		2	3	4	5	6	9	10				2	3	4	5	6	9	10
1.	45									14.	96							
2.	60									15.	65							
3.	18									16.	42							
4.	36									17.	38							
5.	50									18.	246							
6.	64									19.	501							
7.	40									20.	160							
8.	39									21.	432							
9.	110									22.	124							
10.	75									23.	87							
11.	90									24.	705							
12.	51									25.	402							
13.	80									26.	120							

Fractions: Multiplication

Multiplying fractions

To **multiply fractions**, multiply the numerators and then multiply the denominators. To work with smaller numbers, remove the common factors before multiplying.

Example:

$$\frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{5}}} \bullet \frac{\overset{\overset{1}{\cancel{2}}}{\cancel{10}}}{\underset{\underset{2}{\cancel{4}}}{\cancel{16}}} = \frac{1}{2}$$

The factor 4 is removed from the 4 in the numerator and the 16 in the denominator, leaving 1 and 4 respectively.

The factor 5 is removed from the 5 in the denominator and the 10 in the numerator, leaving 1 and 2 respectively.

The factor 2 can now be removed from the 2 in the numerator and the 4 in the denominator, leaving 1 and 2 respectively.

Since all factors in common have been removed, the two numerators are multiplied together and the two denominators are multiplied together resulting in the final result of $\frac{1}{2}$.

Rule: $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$ if $b \neq 0$ and $d \neq 0$ (Remember, division by 0 is undefined.)

TRY:

$\frac{2}{3} \cdot \frac{9}{16} =$

$\frac{-3}{4} \cdot \frac{1}{2} \cdot \frac{-8}{9} =$

$\frac{3}{5} \cdot \frac{20}{21} =$

$6 \cdot \frac{7}{12} =$ (remember to write 6 as $\frac{6}{1}$)

What is $\frac{3}{5}$ of $\frac{4}{9}$? (HINT: of means multiply)

TRY these as fast as you can.

1) $\frac{1}{3}\bullet\frac{1}{3}$ ______________

2) $\frac{2}{5}\bullet\frac{3}{5}$ ______________

3) $\frac{3}{8}\bullet\frac{4}{8}$ ______________

4) $\frac{5}{6}\bullet\frac{1}{6}$ ______________

5) $\frac{2}{7}\bullet\frac{3}{7}$ ______________

6) $\frac{1}{4}\bullet\frac{3}{4}$ ______________

7) $\frac{5}{8}\bullet\frac{3}{8}$ ______________

8) $-\frac{7}{8}\bullet\frac{1}{8}$ ______________

9) $\frac{7}{11}\bullet\frac{11}{15}$ ______________

10) $\frac{2}{7}\bullet\frac{7}{8}$ ______________

11) $\frac{1}{5}\bullet\frac{1}{5}$ ______________

12) $-\frac{4}{7}\bullet\frac{2}{7}$ ______________

13) $\frac{1}{6}\bullet\frac{3}{5}$ ______________

14) $-\frac{1}{3}\bullet\frac{1}{3}$ ______________

15) $-\frac{2}{7}\bullet\frac{4}{7}$ ______________

16) $\frac{1}{6}\bullet\left(-\frac{4}{7}\right)$ ______________

17) $-\frac{5}{11}\bullet\frac{2}{3}$ ______________

18) $\frac{2}{9}\bullet\left(-\frac{5}{9}\right)$ ______________

19) $\left(-\frac{6}{13}\right)\bullet\frac{5}{6}$ ______________

20) $\left(-\frac{3}{8}\right)\bullet\left(-\frac{1}{8}\right)$ ______________

21) $\frac{5}{11}\bullet\frac{3}{7}$ ______________

22) $\frac{2}{5}\bullet\frac{4}{5}$ ______________

23) $\frac{5}{9}\bullet\left(-\frac{2}{9}\right)$ ______________

24) $\left(-\frac{6}{7}\right)\bullet\frac{2}{7}$ ______________

25) $\left(-\frac{1}{5}\right)\bullet\left(-\frac{1}{5}\right)$ ______________

26) $\left(-\frac{3}{4}\right)\bullet\frac{1}{4}$ ______________

27) $\frac{3}{8}\bullet\left(-\frac{4}{9}\right)$ ______________

28) $\frac{5}{13}\bullet\frac{13}{15}$ ______________

How did you do?

Fractions: Reciprocal and Division

Vocabulary

Reciprocal	The **reciprocal** of a number, $\frac{a}{b}$ is $\frac{b}{a}$. To form the reciprocal, invert or interchange the numerator and the denominator. When the reciprocal of any number (except 0) is multiplied together with the number, the result is 1.

Example: The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. When multiplied together, the result is 1. $\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \bullet \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}} = \frac{1}{1} = 1$

The reciprocal of $\frac{7}{5}$ is $\frac{5}{7}$.

TRY: What is the reciprocal of $\frac{3}{5}$? _____ What is the reciprocal of $\frac{8}{3}$? _____

Dividing Fractions

To **divide fractions**, multiply the first fraction by the reciprocal of the second fraction.

Example: $\frac{2}{3} \div \frac{5}{7} = ?$ $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \cdot \frac{7}{5} = \frac{14}{15}$

$\frac{3}{8} \div \frac{21}{22} = ?$ $\frac{3}{8} \div \frac{21}{22} = \frac{3}{8} \cdot \frac{22}{21} = \frac{\overset{1}{\cancel{3}}}{\underset{4}{\cancel{8}}} \cdot \frac{\overset{11}{\cancel{22}}}{\underset{7}{\cancel{21}}} = \frac{11}{28}$

Rule: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$ as long as $b \neq 0$, $c \neq 0$, and $d \neq 0$ (Remember, division by 0 is undefined.)

TRY: $\frac{3}{5} \div \frac{7}{10} =$ $\frac{2}{7} \div \frac{8}{21} =$ $\frac{-4}{9} \div \frac{-16}{45} =$

Since the fraction bar means division, $\frac{2}{7} \div \frac{8}{21}$ could be thought of as the **complex fraction**: $\frac{\frac{2}{7}}{\frac{8}{21}}$

When working with a complex fraction, write it first as the fraction in the numerator divided by the fraction in the denominator. Then, rewrite it as the first fraction times the reciprocal of the second.

Algebra: Multiplication Property of Equality

Brief review: The **reciprocal** of a number, $\frac{a}{b}$ is $\frac{b}{a}$.

When the reciprocal of any number (except 0) is multiplied together with the number, the result is 1.

What is the reciprocal of: $\frac{2}{5}$ $\frac{-3}{7}$ $\frac{8}{11}$ -9

What is: $\frac{2}{5} \bullet \frac{5}{2}$ $\frac{-3}{7} \bullet \frac{-7}{3}$ $\frac{8}{11} \bullet \frac{11}{8}$ $-9 \bullet \frac{-1}{9}$

Multiplication property of Equality Multiplying both sides of an equation by the same number, does not change the solution of the equation.	If $a = b$, then $a \bullet c = b \bullet c$	If $5x = 20$, then $\frac{1}{5} \bullet 5x = \frac{1}{5} \bullet 20$

Finish the example: $5x = 20$ Multiply by the reciprocal of 5 to isolate the x.

Division property of Equality Dividing both sides of an equation by the same number, does not change the solution set of the equation.	If $a = b$, then $\frac{a}{c} = \frac{b}{c}$ $(c \neq 0)$	If $-4y = 12$, then $\frac{-4y}{-4} = \frac{12}{-4}$

Finish the example: $-4y = 12$ Divide by the coefficient to isolate the y.

TRY: (use either property)

$7x = -21$ $5x + 3x = 32$ (don't forget to combine like terms first)

Fractions: Addition and Subtraction

Adding or Subtracting Fractions with like denominators

To **add or subtract fractions**, be sure the denominators are alike then combine the numerators following the steps for adding or subtracting integers.

Example: $\frac{1}{7}+\frac{5}{7}=\frac{6}{7}$ $\frac{5}{9}-\frac{2}{9}=\frac{3}{9}=\frac{1}{3}$ Be sure to reduce

TRY:

1) $\frac{3}{8}+\frac{4}{8}$ ______________

2) $\frac{5}{6}-\frac{1}{6}$ ______________

3) $\frac{1}{4}-\frac{3}{4}$ ______________

4) $\frac{5}{8}+\frac{3}{8}$ ______________

5) $-\frac{7}{8}+\frac{2}{8}$ ______________

6) $\frac{7}{11}-\frac{5}{11}$ ______________

7) $\frac{2}{9}+\frac{4}{9}$ ______________

8) $-\frac{4}{7}+\frac{2}{7}$ ______________

9) $\frac{1}{6}-\frac{5}{6}$ ______________

10) $-\frac{1}{3}+\frac{1}{3}$ ______________

11) $-\frac{2}{7}+\frac{4}{7}$ ______________

12) $\frac{5}{11}-\frac{7}{11}$ ______________

13) $-\frac{2}{9}+\frac{5}{9}$ ______________

14) $\frac{3}{8}-\frac{7}{8}$ ______________

15) $\frac{5}{11}+\frac{3}{11}$ ______________

16) $\frac{2}{5}-\frac{4}{5}$ ______________

17) $\frac{5}{9}-\frac{2}{9}$ ______________

18) $\frac{2}{7}-\frac{6}{7}$ ______________

19) $-\frac{1}{5}-\frac{1}{5}$ ______________

20) $-\frac{3}{4}+\frac{1}{4}$ ______________

Vocabulary

Common Multiple	A number that is evenly divisible by all the numbers in a given group of numbers
Least Common Multiple	The smallest number of the common multiples of a given group of numbers.

What is the smallest number that is a multiple of 4 and 6? The number 24 is a multiple of both 4 and 6, but so is 12. One could list all the multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40 …
And then list all the multiples of 6: 6, 12, 18, 24, 30, 36, 42 … And then compare the two lists and discover that 12 is the smallest number in common.
While this might work for fairly small numbers, it is a time-consuming method for larger numbers.

The easiest way to find the smallest multiple is to factor each number and make a **factor-tree table**.

1. Write each number as a product of prime factors on a separate line in the table.
   $4 = 2 \cdot 2$ $6 = 2 \cdot 3$
2. Line up the prime factors of the second number under the same prime factors of the first number.
3. The least common multiple will be the product formed by multiplying together a factor from **every** column.

4 :	2	2		
6 :	2		3	Put 3 in a different column because it is different than any factor in 4.
LCM:	2	2	3	The LCM is 2 · 2 · 3 or 12.

<u>Adding or Subtracting Fractions with unlike denominators</u>

To add or subtract fractions with unlike denominators, one must first determine the **least common multiple** (LCM) and then convert each fraction into an equivalent fraction with that least common multiple as the denominator before adding the numerators.

Add: $\frac{3}{4}+\frac{1}{6}$ We have already determined that the LCM of 4 and 6 is 12.

So, to add $\frac{3}{4}+\frac{1}{6}$, convert each fraction into an equivalent fraction with the LCM as the common denominator.

[The smallest multiple of two numbers is known as the LCM. The smallest multiple of two denominators is known as the LCD (Least Common Denominator). These two terms are interchanged rather frequently.]

$\frac{3}{4}=\frac{3\cdot 3}{4\cdot 3}=\frac{9}{12}$	What times 4 gives 12? 3. So multiply both numerator and denominator by 3. If you don't know the answer (when you are working with large numbers), the value to use is the factor(s) that appear in the LCM that are not in the denominator given. Since, 3 is the factor in the factor-tree table that appears in the LCM of 12, but is not in a factor in the 4 row, multiply both the numerator and the denominator of the first fraction by 3.
$\frac{1}{6}=\frac{1\cdot 2}{6\cdot 2}=\frac{2}{12}$	What times 6 gives 12? 2. So multiply both numerator and denominator by 2. Since 2 is the factor in the factor-tree table that appears in the LCM of 12, but is not in the 6 row, multiply both the numerator and denominator of the second fraction by 2.
$\frac{3}{4}+\frac{1}{6}=\frac{9}{12}+\frac{2}{12}=\frac{11}{12}$	Now, add the two fractions with common denominators. (Remember, just add the numerators of the two equivalent fractions to get the final answer).

Summary: $\frac{3}{4}+\frac{1}{6}=\frac{9}{12}+\frac{2}{12}=\frac{11}{12}$ 12 is the LCD.

One side benefit of using a factor-tree table is that it also shows the common factors one can use to reduce a fraction. For example, since 2 appears in the table in both the 4 row and in the 6 row, 2 can be used to reduce a fraction such as $\frac{4}{6}$. Remember, one can reduce a fraction by removing the common factors. $\frac{4}{6}=\frac{2\cdot 2}{2\cdot 3}=\frac{2}{3}$

TRY:

1) $\frac{3}{5}+\frac{2}{3}$ ________

2) $\frac{1}{9}-\frac{5}{6}$ ________

3) $\frac{5}{6}+\frac{3}{8}$ ________

4) $-\frac{7}{8}+\frac{3}{4}$ ________

5) $-\frac{4}{7}+\frac{3}{8}$ ________

6) $-\frac{1}{3}+\frac{2}{5}$ ________

7) $-\frac{3}{4}+\frac{5}{6}$ ________

8) $-\frac{2}{9}+\frac{5}{6}$ ________

To arrange fractions from smallest to largest in value, convert all fractions to equivalent fractions with the same common denominator and then arrange the fractions based on the numerators.

Arrange: $\frac{3}{4}, \frac{-7}{12}, \frac{-5}{6}, \frac{7}{18}$ The LCD is 36. Equivalent fractions are: $\frac{27}{36}, \frac{-21}{36}, \frac{-30}{36}, \frac{14}{36}$

Arranged in order: $\frac{-30}{36}, \frac{-21}{36}, \frac{14}{36}, \frac{27}{36}$

TRY: Arrange in order: $\frac{2}{15}, \frac{-2}{5}, \frac{-5}{9}, \frac{4}{45}$

Fractions: Operations on Mixed Numbers

Vocabulary

Mixed Number	The sum of a whole number and a proper fraction Example: $2\frac{3}{4}$

An improper fraction can always be written as a mixed number or a whole number (if the denominator is a factor of the numerator).

To convert from an improper fraction to a mixed number:

$\frac{11}{4}$ $4\overline{)11}$ $2, r = 3$	Divide the numerator by the denominator. If there is a remainder, write the remainder over the original denominator. 11 divided by 4 is 2 with a remainder of 3.
$2\frac{3}{4}$	The mixed number is composed of the integer quotient as the whole number part and the remainder, if any, as the numerator of the proper fraction part.

TRY:

Convert each of the following improper fractions to mixed numbers.

$\frac{17}{3}$ $\frac{-19}{4}$ $\frac{-20}{5}$ $\frac{23}{5}$

To convert from a mixed number to a improper fraction:

$2\frac{3}{4}$	Multiply the whole number by the denominator.
$(2 \bullet 4) + 3$	Add the numerator of the fraction to that product.
$\frac{11}{4}$	Write that sum over the original denominator.

TRY:

Convert each of the following mixed numbers to improper fractions.

$4\frac{3}{5}$ $-3\frac{1}{7}$ -6 $5\frac{3}{4}$

To MULTIPLY two mixed numbers:

$2\frac{3}{4}\bullet 1\frac{5}{33}$	Change each mixed number to improper fractions.
$\frac{11}{4}\bullet\frac{38}{33}$	Simplify if possible. $\frac{11}{2\cdot 2}\bullet\frac{2\cdot 19}{3\cdot 11}$
$\frac{1}{2}\bullet\frac{19}{3}$	Multiply the resulting fractions.
$\frac{19}{6}$	

CAUTION:

$2\frac{3}{4}\bullet 1\frac{1}{7}\neq(2\bullet 1)+(\frac{3}{4}\bullet\frac{1}{7})$ Be sure to convert each mixed number to improper fractions first!

$$2\frac{3}{4}\bullet 1\frac{1}{7}=\frac{11}{4}\bullet\frac{8}{7}=\frac{11}{1}\bullet\frac{2}{7}=\frac{22}{7}$$

TRY:

Multiply.

$4\frac{1}{5}\bullet\frac{4}{7}$

$-3\frac{1}{7}\bullet 2\frac{8}{11}$

To DIVIDE two mixed numbers:

Change each mixed number to improper fractions and follow the rules for dividing fractions.

$$5\frac{3}{5}\div 2\frac{1}{10} \rightarrow \frac{28}{5}\div\frac{21}{10} \rightarrow \frac{28}{5}\bullet\frac{10}{21} \rightarrow \frac{4\cdot 7}{5\cdot 1}\bullet\frac{5\cdot 2}{3\cdot 7} \rightarrow \frac{4}{1}\bullet\frac{2}{3} \rightarrow \frac{8}{3} \rightarrow 2\frac{2}{3}$$

TRY:

Divide.

$-3\frac{1}{7}\div 2\frac{2}{21}$

$-4\frac{1}{5}\div 3$

To multiply or divide a mixed number and whole number, write the whole number as an improper fraction first.

To ADD two mixed numbers:

Change each mixed number to improper fractions and follow the rules for adding fractions.

$$-5\frac{3}{5}+2\frac{1}{10} \rightarrow \frac{-28}{5}+\frac{21}{10} \rightarrow \frac{-56}{10}+\frac{21}{10} \rightarrow \frac{-35}{10} \rightarrow -3\frac{5}{10} \rightarrow -3\frac{1}{2}$$

TRY:

Add.

$$-3\frac{1}{7}+2\frac{3}{4}+1\frac{3}{14}$$

To SUBTRACT two mixed numbers:

Change each mixed number to improper fractions and follow the rules for subtracting fractions.

$$5\frac{3}{5}-2\frac{1}{10} \rightarrow \frac{28}{5}-\frac{21}{10} \rightarrow \frac{56}{10}-\frac{21}{10} \rightarrow \frac{56}{10}+\frac{-21}{10} \rightarrow \frac{35}{10} \rightarrow 3\frac{5}{10} \rightarrow 3\frac{1}{2}$$

$$5-2\frac{1}{10} \rightarrow \frac{25}{5}-\frac{21}{10} \rightarrow \frac{50}{10}-\frac{21}{10} \rightarrow \frac{50}{10}+\frac{-21}{10} \rightarrow \frac{29}{10} \rightarrow 2\frac{9}{10}$$

TRY:

Subtract.

$$3\frac{1}{7}-2\frac{2}{21}$$

$$-4-3\frac{2}{7}$$

Fractions: Complex Fractions

Earlier, we saw that the fraction bar means division: $\frac{2}{7} \div \frac{8}{21}$could be thought of as the **complex fraction**: $\dfrac{\frac{2}{7}}{\frac{8}{21}}$

Vocabulary

Complex fraction	A fraction with a fraction in the numerator and a fraction in the denominator.

To simplify a complex fraction, write the fraction as a division problem and follow the rules for dividing fractions.

$$\dfrac{\frac{2}{7}}{\frac{8}{21}} \rightarrow \frac{2}{7} \div \frac{8}{21} \rightarrow \frac{2}{7} \bullet \frac{21}{8} \rightarrow \frac{2}{7} \bullet \frac{3 \cdot 7}{4 \cdot 2} \rightarrow \frac{1}{1} \bullet \frac{3}{4} \rightarrow \frac{3}{4}$$

If the numerator or the denominator of the complex fraction is composed of something more than a single fraction, follow the rules for forming a single fraction in the numerator and in the denominator, then simplify.

$$\dfrac{4 + \frac{2}{7}}{\frac{2}{3} - \frac{8}{21}} \rightarrow \dfrac{\frac{28}{7} + \frac{2}{7}}{\frac{14}{21} - \frac{8}{21}} \rightarrow \dfrac{\frac{30}{7}}{\frac{6}{21}} \rightarrow \frac{30}{7} \bullet \frac{21}{6} \rightarrow \frac{5 \cdot 6}{7} \bullet \frac{3 \cdot 7}{6} \rightarrow \frac{5}{1} \bullet \frac{3}{1} \rightarrow 15$$

TRY:

$$\dfrac{\frac{3}{7}}{\frac{5}{14}}$$

$$\dfrac{\frac{2}{5} + \frac{2}{3}}{3 - 1\frac{2}{5}}$$

Fractions in Applications

Fractions appear in many applied problems. Read each of the following problems carefully. Create the equation needed for solving the problem. Check your answers with the Lesson.

1. JJ has 23 gallons of fertilizer. If 4 gallons are needed to cover a football field, how many fields can JJ cover?

2. A recipe for cookies calls for $\frac{3}{4}$ cup of flour. How much flour will be needed to make 6 batches of cookies?

3. A patio requires $2\frac{1}{6}yd^3$ of concrete to cover it. If CJ wants to enlarge the patio to $1\frac{2}{3}$ times its current size, how much concrete will be needed to cover the whole patio?

4. In the city, $\frac{2}{3}$ of the people surveyed drive to work. Of those, $\frac{2}{7}$ drive a truck. What fraction of those surveyed drive a truck?

5. A piece of ribbon is $3\frac{1}{5}yd$ long. If it takes $\frac{4}{5}yd$ to make a bow. How many bows can be made from the ribbon?

6. If the jug has $4\frac{2}{3}$ gallons of water and $2\frac{3}{4}$ gallons are poured out, how much water is left in the jug?

7. What is the perimeter (distance all the way around) of a rectangular frame that is $\frac{3}{4}ft$ long and $\frac{2}{3}ft$ wide? What is the area (length times width) enclosed by the frame?

Algebra: Linear Equations in One Variable

To solve a linear equation:

1. Use the distributive property to remove any parentheses.
2. Combine any like terms.
3. Use the addition (or subtraction) property of equality to move all constants to one side of the equation and all terms with variables to the other side of the equation.
4. Use the multiplication or division property of equality to isolate the variable and reveal the solution.
5. Check your solution by replacing the variable in the original equation with your solution.

Example: Solve $5x = 2x - 12$ for x.

$5x = 2x - 12$ $5x - 2x = 2x - 2x - 12$	Use the subtraction property of equality to move the $2x$ to the left side to get the variable terms on the same side.
$3x = -12$ $\frac{3x}{3} = \frac{-12}{3}$	Combine like terms. Use the division property of equality to divide both sides by 3 to isolate the variable.
$x = -4$	Simplify. Check the solution. $5(-4) = 2(-4) - 12$; $-20 = -8 - 12$; $-20 = -20$ is true. The solution is {-4}

Example: Solve $\frac{2}{5}x - 7 = 9$ for x.

$\frac{2}{5}x - 7 = 9$ $\frac{2}{5}x - 7 + 7 = 9 + 7$	Use the addition property of equality to move the -7 to the right side to isolate the variable term.
$\frac{2}{5}x = 16$ $\frac{5}{2} \bullet \frac{2}{5}x = \frac{5}{2} \bullet 16$	Combine like terms. Use the multiplication property of equality to multiply both sides by the reciprocal to isolate the variable. Reduce.
$1 \bullet x = 5 \bullet 8$ $x = 40$	Simplify. Check the solution. $\frac{2}{5}(40) - 7 = 9$; $2 \bullet 8 - 7 = 9$; $16 - 7 = 9$; $9 = 9$ is true. The solution is {40}

TRY: Solve each of the following for the variable.

$3x-5=7$

$5-6x=-19$

$\frac{2}{5}x+7=17$

$8x-6=4x+18$

$-3(2x+4)=-10x$

$4(-x-2)=-2(2x+7)-5$

Fractions in Equations

Some think working with fractions in equations is much easier than working with fractions in expressions. (I do.)

To prepare the equation, just multiply every term in the equation by the Least Common Denominator of all the fractions. If you select the correct LCD, all the denominators will simplify to 1 and the equation will no longer contain fractions!

Example: Solve $\frac{3}{8}x = 6$ for x.

$\frac{3}{8}x = 6$ $8(\frac{3}{8}x) = 8(6)$	Multiply both sides by 8. Simplify.
$3x = 48$ $\frac{3x}{3} = \frac{48}{3}$	Use the division property to isolate the variable. Simplify.
$x = 16$	Check the solution. $\frac{3}{8}(16) = 6;\ 3(2) = 6;\ 6 = 6$ is true. The solution is: $\{16\}$

Example: Solve $\frac{1}{2}x - 1 = \frac{1}{8}x + \frac{7}{8}$ for x.

$\frac{1}{2}x - 1 = \frac{1}{8}x + \frac{7}{8}$	Since there are multiple fractions, multiply each term by the LCD – even the terms without fractions. The LCD is 8.
$8 \cdot \frac{1}{2}x - 8 \cdot 1 = 8 \cdot \frac{1}{8}x + 8 \cdot \frac{7}{8}$	Simplify. $8 \cdot \frac{1}{2}x - 8 \cdot 1 = 8 \cdot \frac{1}{8}x + 8 \cdot \frac{7}{8};\ 4x - 8 = 1x + 7$
$4x - 8 = 1x + 7$	Use the addition or subtraction property to combine like variables on one side and the constants on the other.
$4x - 8 + 8 = 1x + 7 + 8$ $4x = 1x + 15$ $4x - 1x = 1x - 1x + 15$	Simplify.
$3x = 15$	Use the division property of equality to isolate the variable.
$\frac{3x}{3} = \frac{15}{3}$	Simplify. $\frac{3x}{3} = \frac{15}{3};\ x = 5$
$x = 5$	Check the solution. $\frac{1}{2} \cdot 5 - 1 = \frac{1}{8} \cdot 5 + \frac{7}{8};\quad \frac{5}{2} - 1 = \frac{5}{8} + \frac{7}{8};\quad \frac{5}{2} \cdot 8 - 1 \cdot 8 = \frac{5}{8} \cdot 8 + \frac{7}{8} \cdot 8;$ $5 \cdot 4 - 8 = 5 + 7;\quad 20 - 8 = 5 + 7\ ;\quad 12 = 12$ is true. The solution is: $\{5\}$

TRY:

$\frac{2}{3}x = -10$

$\frac{7}{4}x = 42$

$\frac{1}{3}x = \frac{11}{15}$ (This could be written as $\frac{x}{3} = \frac{11}{15}$)

$-\frac{4}{9}x = -\frac{5}{6}$

$\frac{4}{9}x - 11 = 1$

$\frac{5}{3}x + 6 = 41$

When more than one term contains a fraction, multiply ALL terms by the common denominator.

$\frac{1}{5}x + 1 = \frac{3}{10}x + \frac{1}{4}$

$\frac{2}{9}x - \frac{1}{2} = \frac{1}{18}x + \frac{2}{3}$

Algebra: Applications of Linear Equations in One Variable (Overview)

Steps to Solving Applied Problems

Every day we encounter application or word problems. There is **NO** standard procedure for solving these problems, but there are some guidelines that can be used.

1. **READ** the problem until you understand the problem. Determine what information is given and what you are asked to find. Try to guess what the answer might be.
   DRAW a picture, make a diagram, or construct a table to help illustrate the problem.

2. **IDENTIFY** what you are being asked to find.
   SELECT a variable to represent one of the unknowns.
   WRITE down what the variable represents.
   DEFINE any other unknowns in terms of that variable.
   LABEL the picture or diagram or parts of the table with the variable, any other unknowns, and any additional information provided by the problem.

3. **TRANSLATE** the word problem into an equation that models or represents the situation.
   RESTATE the problem in your own words.
   SEPARATE the larger word problem into small parts.
   WRITE an algebraic expression to represent each part.
   CREATE an algebraic equation that represents the situation by combining the expressions.

4. **SOLVE** the equation.

5. **INTERPRET** the meaning of your solution in terms of the original situation.
   VERIFY that your solution answers the question posed in the original problem and makes sense.
   CHECK your answer by using it to solve the original problem (not the equation).
   FIND other unknowns if necessary.

6. **STATE** your solution in a sentence including appropriate labels as necessary.

Phrases often used in Number problems.

The sum of two *consecutive* integers is written: $x+(x+1)$

The sum of three consecutive *even* integers is written: $x+(x+2)+(x+4)$

The sum of three consecutive *odd* integers is written: $x+(x+2)+(x+4)$

Situation statement:

Four plus the sum of two consecutive odd integers is one less than three times the first odd integer.

State the Equation:

$$4+[x+(x+2)]=3x-1$$

Solve:

$$4+[x+(x+2)]=3x-1$$
$$4+[2x+2]=3x-1$$
$$2x+6=3x-1$$
$$2x-2x+6=3x-2x-1$$
$$6=x-1$$
$$6+1=x-1+1$$

$7=x$ The two numbers are 7 and 9.

Check:

$$4+(7+9)=3(7)-1$$
$$4+16=21-1$$
$$20=20$$

TRY:

Seven more than one-third of a number is 10.

What is the equation?

What is the solution?

General Quantities

A. Dr. Jay has a total of 48 students in his two labs. There are 4 more students in his 8:00 a.m. lab than in his 4:00 p.m. lab. Find the number of students in each lab.

Unknowns: S = number of students in the 4:00 p.m. lab.
S+4 = number of students in the 8:00 a.m. lab

Equation: S + (S+4) = 48

B. There were three times more women participating in the Victory Run than men. A total of 60 people participated in the Run. Find the number of men and the number of women that participated.

HINT: express 'women' in terms of 'men'

Unknowns: M = number of men
3M = number of women

Equation: M + 3M = 60

TRY:

Fido has a total of 62 treats. There are 14 more soft chewy treats than there are hard crunchy treats. Find the number of each type of treat. HINT: express 'chewy' in terms of 'hard'

There are 91 students registered in 3 sections of algebra. There are twice as many students in Section A as in Section B and 11 more in Section C than in Section B. How many students are in each section? HINT: express 'A' and 'C' in terms of 'B'

Mrs. J. is 12 years more than 5 times the age of her daughter. If the difference between their ages is 24, how old is Mrs. J? HINT: express 'Mrs. J' in terms of 'daughter'

<u>Different Lengths</u>

A. A plumber has a pipe that is 21 feet long. He needs to cut it into two sections so that one section is half as long as the other. Find the length of each section of pipe.

Unknowns: F = length in feet of the longer pipe

$\frac{1}{2}F$= length in feet of the shorter pipe

Equation: $F+\frac{1}{2}F=21$

B. A 24-foot chain must be cut into three pieces. The longest piece needs to be three times the length of the shortest piece. The medium-length piece needs to be twice the length of the shortest piece. Find the lengths of each piece.

Unknowns: C = length in feet of the shortest piece
2C= length in feet of the medium-length piece
3C= length in feet of the longest piece

Equation: C + 2C + 3C = 24

TRY:

For Ky's art project, a cord must be cut into two pieces. The longer piece needs to be three times the length of the shorter piece. If the full length of the cord is 64 feet, find the lengths of the two pieces.

Consecutive Integers

A. The sum of three consecutive integers is 123. Find the three integers.

Unknowns: X = first integer
X+1 = second integer
X+2 = third integer

Equation: X + (X+1) + (X+2) = 123

B. If the smaller of two consecutive odd integers is subtracted from twice the larger one, then the result is 13. Find the smaller odd integer.

Unknowns: N = first odd integer
N+2 = second odd integer

Equation: 2 (N+2) – N = 13

TRY:

If the smaller of two consecutive even integers is added to three times the larger one, then the result is five times the smaller one. Find the two integers.