## Fractions: Addition and Subtraction

## Adding or Subtracting Fractions with like denominators

To **add or subtract fractions**, be sure the denominators are alike then combine the numerators following the steps for adding or subtracting integers.

Example:  $\frac{1}{7} + \frac{5}{7} = \frac{6}{7}$   $\frac{5}{9} - \frac{2}{9} = \frac{3}{9} = \frac{1}{3}$  Be sure to reduce



## Vocabulary

Common Multiple	A number that is evenly divisible by all the numbers in a given group of numbers
Least Common Multiple	The smallest number of the common multiples of a given group of numbers.

What is the smallest number that is a multiple of 4 and 6? The number 24 is a multiple of both 4 and 6, but so is 12. One could list all the multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40 ... And then list all the multiples of 6: 6, 12, 18, 24, 30, 36, 42 ... And then compare the two lists and discover that 12 is the smallest number in common.

While this might work for fairly small numbers, it is a time-consuming method for larger numbers.

The easiest way to find the smallest multiple is to factor each number and make a **factor-tree table**. 1. Write each number as a product of prime factors on a separate line in the table.

$$4 = 2 \cdot 2 \qquad 6 = 2 \cdot 3$$

- 2. Line up the prime factors of the second number under the same prime factors of the first number.
- 3. The least common multiple will be the product formed by multiplying together a factor from **every** column.

4 :	2	2		
6:	2		3	Put 3 in a different column because it is different than any factor in 4.
LCM:	2	2	3	The LCM is 2 · 2 · 3 or 12.

## Adding or Subtracting Fractions with unlike denominators

To add or subtract fractions with unlike denominators, one must first determine the **least common multiple** (LCM) and then convert each fraction into an equivalent fraction with that least common multiple as the denominator before adding the numerators.

Add: 
$$\frac{3}{4} + \frac{1}{6}$$
 We have already determined that the LCM of 4 and 6 is 12.  
So, to add  $\frac{3}{4} + \frac{1}{6}$ , convert each fraction into an equivalent fraction with the LCM as the common denominator.

[The smallest multiple of two numbers is known as the LCM. The smallest multiple of two denominators is known as the LCD (Least Common Denominator). These two terms are interchanged rather frequently.]

$\frac{3}{4} = \frac{3 \cdot 3}{4 \cdot 3} = \frac{9}{12}$	What times 4 gives 12? 3. So multiply both numerator and denominator by 3. If you don't know the answer (when you are working with large numbers), the value to use is the factor(s) that appear in the LCM that are not in the denominator given. Since, 3 is the factor in the factor-tree table that appears in the LCM of 12, but is not in a factor in the 4 row, multiply both the numerator and the denominator of the first fraction by 3.
$\frac{1}{6} = \frac{1 \cdot 2}{6 \cdot 2} = \frac{2}{12}$	What times 6 gives 12? 2. So multiply both numerator and denominator by 2. Since 2 is the factor in the factor-tree table that appears in the LCM of 12, but is not in the 6 row, multiply both the numerator and denominator of the second fraction by 2.
$\frac{3}{4} + \frac{1}{6} = \frac{9}{12} + \frac{2}{12} = \frac{11}{12}$	Now, add the two fractions with common denominators. (Remember, just add the numerators of the two equivalent fractions to get the final answer).

Summary: 
$$\frac{3}{4} + \frac{1}{6} = \frac{9}{12} + \frac{2}{12} = \frac{11}{12}$$
 12 is the LCD.

One side benefit of using a factor-tree table is that it also shows the common factors one can use to reduce a fraction. For example, since 2 appears in the table in both the 4 row and in the 6 row, 2 can be used to reduce a fraction such as  $\frac{4}{6}$ . Remember, one can reduce a fraction by removing the common factors.  $\frac{4}{6} = \frac{2 \cdot 2}{2 \cdot 3} = \frac{2}{3}$ 

TRY:



To arrange fractions from smallest to largest in value, convert all fractions to equivalent fractions with the same common denominator and then arrange the fractions based on the numerators.

Arrange:  $\frac{3}{4}, \frac{-7}{12}, \frac{-5}{6}, \frac{7}{18}$  The LCD is 36. Equivalent fractions are:  $\frac{27}{36}, \frac{-21}{36}, \frac{-30}{36}, \frac{14}{36}$ Arranged in order:  $\frac{-30}{36}, \frac{-21}{36}, \frac{14}{36}, \frac{27}{36}$ TRY: Arrange in order:  $\frac{2}{15}, \frac{-2}{5}, \frac{-5}{9}, \frac{4}{45}$