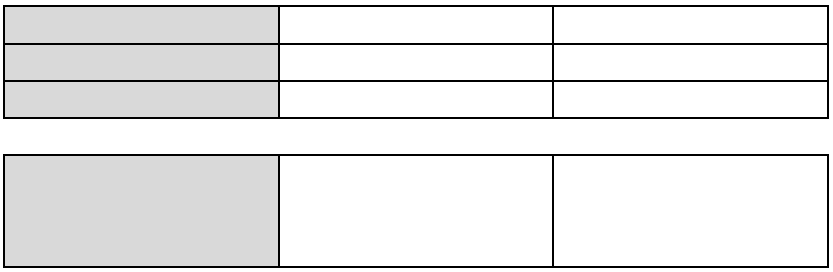
Fractions: Equivalent Fractions

Vocabulary

Equivalent Fractions	Two fractions representing the same portion.



$\frac{3}{9}$ represents the same portion as $\frac{1}{3}$ and are said to be equivalent fractions.

Vocabulary

Cross Product	Given two fractions: $\frac{a}{b}=\frac{c}{d}$, $a \bullet d$ and $b \bullet c$ are cross products.

Test for Equivalent Fractions: Two fractions are equivalent if the cross products are equal.

Are $\frac{3}{9}$ and $\frac{1}{3}$ equivalent fractions? Does $3 \cdot 3 = 9 \cdot 1$? Yes. They are equivalent fractions.

TRY:

Are $\frac{2}{7}$ and $\frac{6}{21}$ equivalent fractions?

Are $\frac{2}{5}$ and $\frac{9}{15}$ equivalent fractions?

Fundamental Principles of Fractions

$\frac{a}{b}=\frac{a \div c}{b \div c}$ for the fraction $\frac{a}{b}$ and any nonzero number c. As long as we divide both the numerator and the denominator by the same number, the result is an equivalent fraction.
$\frac{a}{b}=\frac{a \bullet c}{b \bullet c}$ for the fraction $\frac{a}{b}$ and any nonzero number c. As long as we multiply both the numerator and the denominator by the same number, the result is an equivalent fraction.

Vocabulary

Lowest terms or Simplest form	When the numerator and the denominator have no common factors other than the number 1, the fraction is said to be in lowest terms.

Reducing Fractions

To change, or reduce, a fraction into lowest terms, one writes the numerator and denominator as the product of their primes and then divides the numerator and the denominator by the same common factor.

$$\frac{8}{12}=\frac{2*\cancel{2}*\cancel{2}}{3*\cancel{2}*\cancel{2}}=\frac{2}{3}$$

$\frac{12}{8}$ is an *improper* fraction. It may be written as an improper fraction in lowest terms as $\frac{3}{2}$ or as a mixed number as $1\frac{1}{2}$. In either case, the proper form is in lowest terms. $\frac{3}{1}$ is an improper fraction. The correct form is just 3.

TRY: Reduce the following: $\frac{25}{5}=$ $\frac{49}{63}=$ $\frac{-36}{-24}=$

Making equivalent fractions

To make an equivalent fraction with a different denominator, one multiplies the numerator and the denominator of the original fraction by the same factor.

$\frac{3}{7}=\frac{?}{28}$ To form the denominator 28, the 7 was multiplied by 4.

$\frac{3}{7}=\frac{3\cdot 4}{7\cdot 4}=\frac{12}{28}$ To create the equivalent fraction, multiply the numerator by 4 as well.

TRY: Form equivalent fractions

$\frac{5}{9}=\frac{?}{36}$ $\frac{2}{11}=\frac{?}{66}$ $\frac{-3}{13}=\frac{?}{39}$

Vocabulary

Standard form	The standard form of a negative fraction is with the negative sign in the numerator or in front of the entire fraction. When used for evaluating an expression, the negative sign is considered with the numerator. $\frac{2}{-5}$ in standard form is $\frac{-2}{5}$ or $-\frac{2}{5}$

Divisibility Rules can help one quickly spot common factors.

A number is divisible by	**Example**
…2 if it is an even number.	7394 is divisible by 2 because it ends in 4, making it an even number.
…3 if the sum of its digits is divisible by 3.	837 -- Add the digits: 8 + 3 + 7 = 18. Since 18 is divisible by 3, the number 837 is divisible by 3.
…4 if its last two digits form a number that is divisible by 4.	5932 -- The last two digits form the number 32. Since 32 is divisible by 4, the number 5932 is divisible by 4.
…5 if the number ends in 0 or 5.	645 is divisible by 5 since it ends in 5.
…6 if it divisible by 2 and 3.	1248 – The number is divisible by 2 since it is an even number. The number is divisible by 3 since the sum of its digits is divisible by 3: 1 + 2 + 4 + 8 = 15. Therefore, the number 1248 is divisible by 6.
…9 if the sum of its digits is divisible by 9.	837 -- Add its digits: 8 + 3 + 7 = 18. Since 18 is divisible by 9, the number 837 is divisible by 9.
…10 if it ends in a zero.	890 is divisible by 10 because it ends in 0.

Determine if each of the following numbers is divisible by 2, 3, 4, 5, 6, 9, and/or 10.
(Mark the column with an X.)

		2	3	4	5	6	9	10				2	3	4	5	6	9	10
1.	45									14.	96							
2.	60									15.	65							
3.	18									16.	42							
4.	36									17.	38							
5.	50									18.	246							
6.	64									19.	501							
7.	40									20.	160							
8.	39									21.	432							
9.	110									22.	124							
10.	75									23.	87							
11.	90									24.	705							
12.	51									25.	402							
13.	80									26.	120							