Lesson 04: Factors and Fractions

Factors, Prime Numbers, and GCF

Vocabulary

Factor	Each natural number used to form a product. A factor of a natural number is another natural number that will divide exactly into that number with 0 as the remainder. Since $6 \cdot 4 = 24$, then 6 and 4 are factors of 24. $24 \div 6 = 4$, r 0 and $24 \div 4 = 6$, r 0

Every number has at least two factors (or divisors) – the number itself and the number 1.
Since $7 \cdot 1 = 7$, then 7 and 1 are both considered to be factors of 7.

Vocabulary

Prime	A natural number, other than 1, whose only factors are the number 1 and itself.
Composite	A natural number greater than 1, that is not prime. (i.e. it has factors other than 1 and itself).

The number 1 is considered to be neither prime nor composite.

TRY:

Finish the list of the first ten prime numbers: 2 3 ___ ___ ___ ___ ___ ___ ___ 29

When working with division and other areas of mathematics, it is often helpful to write a number as a product of its prime factors. This is called finding the **prime factorization** of the number or **factoring** a number.

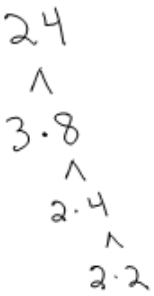
To find all the factors of a given number, one uses a **factor tree**.

Example:

Write 24 as the product of its prime factors.

Two factors of 24 are 6 and 4.
Two factors of 6 are 3 and 2.
Two factors of 4 are 2 and 2.

It doesn't matter which factors one uses to start the factor tree. The results will be the same.



The prime factorization of 24 is
$3 \cdot 2 \cdot 2 \cdot 2$

One continues to build the factor tree until the number at the end of each 'branch' is a prime number.
What about the prime factorization of 48?

The prime factorization of 48 = 2 · 3 · 2 · 2 · 2
Since multiplication is commutative, the order of the factors doesn't matter. We can rearrange the order so all the like factors are together. Then, to condense the result, we can use exponents. Since the 2 appears 4 times, the result can be written as: $48 = 2^4 \cdot 3$.

TRY: What is the prime factorization of 40?

Vocabulary

Common Factor or Divisor	A natural number that is a factor of two other numbers

What are the common factors of 18 and 24?
Staring with 1, list all the factors of 18. Then, list all the factors of 24.

18:	1	2	3		6		9		18		
24:	1	2	3	4	6	8		12		24	
36:	1	2	3	4	6		9	12	18		36
Common:	1	2	3		6						

The common factors of 18, 24, and 36 are: 1, 2, 3, 6

TRY: What are the common factors of 30 and 45?

Vocabulary

Greatest Common Factor (GCF)	The largest factor in common that will divide each of the given numbers exactly.

The common factors of 18, 24, and 36 are: 1, 2, 3, 6
So the GCF of 18, 24, and 36 is 6.

Finding all the factors of three numbers and then finding the greatest common factor takes a lot of time, especially if the numbers are large. There is another way.

Write the prime factorization of each number. (Use factor trees if you wish.)

18 = 2·9 = 2·3·3
24 = 2·12 = 2·2·6 = 2·2·2·3
36 = 6·6 = 2·3·2·3 = 2·2·3·3

Remember, it doesn't matter how you start the process, the result will be the same.

Line up the common factors found. It doesn't matter the order. Just be sure to line up like factors.

18:	2	3	3			The **GREATEST COMMON FACTOR** will be the product of all the common prime factors. If there are no factors in common, the GCF is 1. The **GCF** of 18, 24, and 36 is 2·3 or **6**.
24:	2	3		2	2	
36:	2	3	3	2		
common?	**2**	**3**				

TRY: Find the GCF of 15 and 20. Find the GCF of 12, 24, and 48.

In algebra, it is often helpful to find factors with a given sum or difference. Complete the following.

Find 2 factors of	Whose sum is	**ANSWER**
10	7	5 and 2
28	11	
81	18	
36	13	
24	14	
Find 2 factors of	Whose difference is	**ANSWER**
10	9	10 and 1
18	7	
15	2	
12	1	