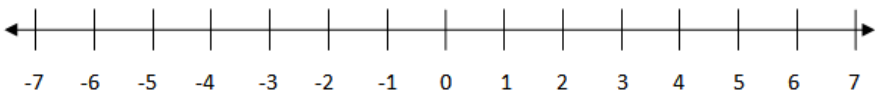
Lesson 2: Integers

Integers, Opposites, and Absolute Value

Vocabulary

Integers	Natural Numbers, their negatives, and zero: … -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, …
Number Line	 Integers are used to name the points shown on a number line.
Negative Integers	Numbers used to name points to the left of 0 on a number line.
Negative sign "-"	The sign used in front of a number to indicate it is a negative number.
Positive Integers	Numbers used to name points to the right of 0 on a number line.
Positive sign "+"	The sign used in front of a number to indicate it is a positive number. If no sign is present, the default value is positive.
Real Numbers	All the points along the number line, including the points between the integers.
Ascending Order	Writing a group of numbers from smallest to largest
Minimum	The least number in a group of numbers
Maximum	The greatest number in a group of numbers
Extreme Values	The least and greatest numbers in a group of numbers

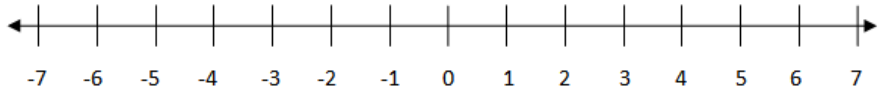
TRY: Given the definitions above, complete the following.

Identify the minimum and maximum values in: -15, 26, -32, -19, 35, 47, -31

Identify which of the following are integers: -19 3,405 -.3 0 ½ -4.8

Vocabulary

Opposite	Two numbers whose points they name are the same distance from 0, but in different directions.



The distance from -4 to 0 along the number line is 4 units. The distance from 4 to 0 along the number line is 4 units. These two distances are the same therefore -4 and 4 are opposites.

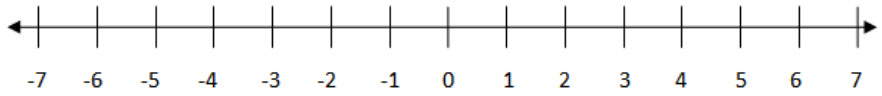
Examples: The opposite of 8 is -8. The opposite of -7 is 7. The opposite of 0 is 0.

Another way of thinking: In your mind, visualize the number line folded in half at the 0 point. Opposite numbers are now opposite from each other. 1 is directly opposite from -1; -5 is directly opposite from 5, and so on.

TRY: The opposite of -3 is ____. The opposite of 10 is _____.

Vocabulary

Absolute Value	The distance on the number line between the point named by a number and 0



The absolute value does not depend on whether the number is to the left or right of 0. It only depends on its distance from 0. The absolute value of a number a is written $|a|$.

Examples:

| 5 | = 5 |3| = 3 | 0 | = 0 - | -8 | = - 8

| -6 | + | 4 | =
6 + 4 = 10

| -14 | - |- 11 | =
14 - 11 = 3

Another way of thinking: Visualize yourself standing on the number line. If you were standing at 5, it would take you 5 steps to reach 0. The absolute value of 5, or |5|, is 5. If you were at -3, it would take you 3 steps to reach 0. The absolute value of -3, or |-3|, is 3. If you were at 0, it would take 0 steps to reach 0. The absolute value of 0, or |0|, is 0.

TRY:

| -7 | = ____ | 8 | = ____

- | -7 | = ____ | 8 | - | -5 | = ____

| -8 | + | 7 | = ____ | -21 | - |- 16 | = ____

| 0 | = ____ | -15 + 4 | = ____

| 8 | - | -8 | = ____ | -9 | - | -9 | = ____

Integers: Addition

Adding with like signs

When adding two integers, **if the signs of the two integers are the same**, add the absolute values of the integers and give the sum the sign of the original integers.

Examples: 5 + 9 = ? 5 added to 9 is 14. The sum will be positive. 5 + 9 = 14

-3 + (-5) = ? Think of this as adding 3 + 5, then give the sum the - sign. -3 + (-5) = -8

Another way of thinking: One can think of adding two integers with same signs as counting how many positive items or how many negative items you have. For example, 5 + 9 would be 5 positive items plus 9 positive items for a total of 14 positive items. -3 + (-5) would be 3 negative items plus 5 negative items for a total of 8 negative items, or -8.

TRY: 7 + 9 = _____ -6 + (-3) = _____

Adding with different signs

When adding two integers, **if the signs of the two integers are different**, subtract the smaller absolute value from the larger absolute value (take the difference), and give the sum the sign of the original integer with the largest absolute value.

Example: -3 + 7 = ? The | -3 | is 3. The | 7 | is 7. The difference between 7 and 3 (on the number line) is 4. Since 7 has the largest absolute value and the sign of the original original 7 was positive, the sign of the answer is positive.

-3 + 7 = 4

8 + (-13) = ? The | 8 | is 8. The |-13| is 13. The difference between 8 and 13 (on the number line) is 5. Since -13 has the largest absolute value and the sign of the original 13 was negative, the sign of the answer is negative.

8 + (-13) = -5

Another way of thinking: One can think of adding two integers with different signs as matching up one negative with one positive and seeing how many of one type is left over. Visualize 8 positively-charged balls and 13 negatively-charged balls being put together. The 8 positively charged balls will match up with 8 of the 13 negatively charged balls. This leaves 5 negatively-charged balls.
So: 8 + (-13) = -5

Yet another way of thinking: One can think of adding two integers with different signs as working with money. I have 8 dollars and now I owe 13 dollars. I am 5 dollars in debt. So: 8 + (-13) = -5

TRY: 6 + (-10) = _____ -5 + 9 = _____

NOTE: It is extremely important for you to be able to quickly add two integers together in your mind.

TRY: Complete the following as quickly as you can.

1) 3 + 9 _____	9) 7 + 3 _____
2) 6 + (-3) _____	10) 9 + 9 _____
3) (-7) + (-7) _____	11) (-3) + (-4) _____
4) (-6) + 8 _____	12) (-6) + 12 _____
5) (-12) + 1 _____	13) (-4) + (-7) _____
6) (-3) + (-8) _____	14) (-5) + 7 _____
7) (-12) + 1 + (-6) + 8 _____	15) (-6) + 12 + 6 + (-3) _____
8) (-4) + (-7) + (-5) + 7 _____	16) (-13) + (-18) + 32 _____

TRY:

DJ has $175 in a checking account. A check was written for $82 and a deposit of $35 was made. What is the resulting balance?

The temperature one morning was -18^{o}. By 3:00 p.m., the temperature had increased by 53^{o}. What was the temperature at 3 p.m.?

Integers: Subtraction

Subtracting two integers

When subtracting two integers, change the subtraction sign to addition and replace the integer being subtracted with its opposite.

DEFINITION: $a - b = a + (-b)$

Examples:

7 - 4 = ? Rewrite it as: 7 + (-4) = ? then follow the rules for addition. 7 + (-4) = 3

-5 - (-3) = ? Rewrite it as: -5 + (+3) = -5 + 3 = ? -5 + 3 = -2

3 - (-4) = ? Rewrite it as: 3 + (+4) = 3 + 4 = ? 3 + 4 = 7

6 - 10 = ? Rewrite it as: 6 + (-10) = ?
The 10 in the original problem is a positive 10. 6 + (-10) = -4

TRY:

1)	7 - 15	_____	9)	-8 - (-5)	_____
2)	-6 - 6	_____	10)	-10 - 6	_____
3)	8 - (-60)	_____	11)	11 - (-5)	_____
4)	44 - 76	_____	12)	3 - (-8)	_____
5)	5 - (-5)	_____	13)	-6 - 12	_____
6)	-6 - 8	_____	14)	-4 - (-7)	_____
7)	5 - 9	_____	15)	34 - (-8)	_____
8)	-12 - 21	_____	16)	-3 - (-8)	_____

TRY:

Hy's checking account shows a balance of $258. It was discovered that a deposit of $65 was accidentally recorded as a check for $65. What is the corrected balance?

Integers: Multiplication

Multiplying with like signs

When multiplying two integers, **if the signs of the two integers are the same**, the product will be **positive**.

A positive integer times a positive integer results in a positive integer.

A negative integer times a negative integer results in a positive integer.

Examples:

$5 \cdot 9 = 45$ $(-3) \cdot (-5) = 15$

TRY: $6 \cdot 8 =$ _____ $(-7) \cdot (-9) =$ _____

Multiplying with different signs

When multiplying two integers, **if the signs of the two integers are different**, the product will be **negative**.

A positive integer times a negative integer results in a negative integer.

A negative integer times a positive integer results in a negative integer.

Examples:

$(-3) \cdot 7 = -21$ $8 \cdot (-5) = -40$

TRY: $(-7) \cdot 6 =$ _____ $8 \cdot (-6) =$ _____

The product of any integer and 0 is 0. Example: $5 \cdot 0 = 0$ $-5 \cdot 0 = 0$

The product of any integer and 1 is that integer. Example: $5 \cdot 1 = 5$ $-5 \cdot 1 = -5$

Caution – Be careful when working with exponents. It makes a difference if the negative sign is included with the base inside the parenthesis or not included.

$$(-3)^2 = (-3) \cdot (-3) = 9 \qquad -3^2 = -(3 \cdot 3) = -9$$

Remember the Order of Operations – it applies to integers as well. **P**lease **E**xcuse **M**y **D**ear **A**unt **S**uz

P E M D A S	1. "P" If parentheses or other grouping symbols such as braces or brackets are present in the expression, evaluate what is in these grouping symbols first. 2. "E" Evaluate all expressions with exponents next. 3. "M, D" Complete any multiplication or division in order, working from left to right. 4. "A, S" Complete any addition or subtraction in order, working from left to right.

Example:	$6 - 3(2^2-7)$	Evaluate the ( ) first. Inside it is an exponent to evaluate. $2^2 = 4$
	$6 - 3(4-7)$	Evaluate the ( ) expression. $(4-7) = -3$
	$6 - 3(-3)$	Complete the multiplication: $3(-3) = -9$
	$6 - (-9)$	Complete the subtraction: $6 - (-9)$ is $6 + (+9) = 15$
	15	Final answer.

TRY:

1) $6 \cdot (-3)$ _____	12) $(-10)^2 \cdot 6$ _____
2) $7 \cdot 6$ _____	13) $11 \cdot (-5)$ _____
3) $(-6) \cdot 6$ _____	14) $8 \cdot 7$ _____
4) $8 \cdot 0$ _____	15) $(-9)^2$ _____
5) $(-7) \cdot (-7)$ _____	16) $-3(-4+8)$ _____
6) $8(-2 - 5)$ _____	17) $-(9)^2$ _____
7) $-5+ (-5)(3)$ _____	18) $(-6) \cdot 12$ _____
8) $(-6) \cdot 8$ _____	19) $(-4) \cdot (-7)$ _____
9) $(-12) \cdot 1$ _____	20) $(-5) \cdot 7$ _____
10) $7 \cdot (-9)$ _____	21) $-6^2 - 2^2$ _____
11) $(-3) \cdot (-8)$ _____	22) $-7^2 - (-2)^2$ _____

Order of Operations is SO important; here are three more to …

TRY:

$3\,(1 + 3)^2 \div 4 \cdot 2 =$

$14 - 8 \div 2 \cdot 3 \div 2 + 2 =$

$10 - 4\,[\,-\,(\,2 + 1\,)\,] \div 2\,(-\,3 + 2\,) =$

Integers: Division

Dividing with like signs

When dividing two integers, **if the signs of the two integers are the same**, the quotient will be **positive**.

A positive integer divided by a positive integer results in a positive number.

A negative integer divided by a negative integer results in a positive number.

Examples:

45 ÷ 9 = 5 (-15) ÷ (-5) = 3

TRY: 48 ÷ 8 = _____ (-56) ÷ (-7) = _____

Dividing with different signs

When dividing two integers, **if the signs of the two integers are different**, the quotient will be **negative**.

A positive integer divided by a negative integer yields a negative number.

A negative integer divided by a positive integer yields a negative number.

Examples:

(-21) ÷ 7 = -3 40 ÷ (-5) = -8

TRY: (-32) ÷ 8 = _____ 54 ÷ (-9) = _____

Remember:

The quotient of any integer and 0 is undefined. 5 ÷ 0 is undefined -5 ÷ 0 is undefined

The quotient of 0 and any integer is 0. 0 ÷ 5 = 0 0 ÷ -5 = 0

The fraction bar serves as a grouping symbol. This means one performs all the operations above the bar separately from the operations below the bar before combining the answer. The fraction bar also means division. So, the final operation to perform after operations have been completed on the top (the numerator) and the bottom (the denominator) is division.

TRY: Evaluate each of the following expressions

$\frac{-60}{5}$ (-18) ÷ (-3) $\frac{54}{9}$ $\frac{56}{-7}$

$\frac{(-5)(-8)}{4}$ $\frac{4+(-16)}{4}$ $\frac{-8+(2)(-5)}{-6-3}$ $\frac{[(-3)^2-1](-4)}{-12+(-4)}$