Lesson 01: Whole Numbers

Whole Numbers and Place Value

Vocabulary

Number System	A way of naming numbers
Digits	Number symbols (numerals) 0, 1, 2, 3, 4, 5, 6, 7,8, 9
Place Value	Position in a number
Decimal place-value system	Value of the position in the number is based on the number 10

Place Value:

Billions Group			Millions Group			Thousands Group			Ones Group		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
7	4	9,	5	0	3,	2	1	8,	9	6	5

Seven hundred forty-nine billion, five hundred three million, two hundred eighteen thousand, nine hundred sixty-five

Standard (numerical) Form: 5,367
Expanded Form: (5 x 1,000) + (3 x 100) + (6 x 10) + (7 x 1)
Word Form: Five thousand, three hundred sixty-seven

Standard (numerical) Form: 400,012,508
Expanded Form: (4 x 100,000,000) + (1 x 10,000) + (2 x 1,000) + (5 x 100) + (8 x 1)
Word Form: Four hundred million, twelve thousand, five hundred eight

Note:

A comma "," is used separate the different groups in the number.
The zero "0" is used as a place holder.
The word "and" is not used in writing the name of this type of number.

TRY:

Complete the following.

Standard Form	
Expanded form	(6 x 10,000) + (2 x 1,000) + (4 x 100) + (3 x 10) + (7 x 1)
Word form	

Standard form	
Expanded form	
Word form	Three million, four hundred fifteen

Whole Numbers: Addition

Vocabulary

Natural Numbers	Commonly called the counting numbers: 1, 2, 3, 4, 5, 6, …
Whole Numbers	Natural Numbers and zero: 0, 1, 2, 3, 4, 5, 6, …
Addition	Combining two or more groups of similar objects together
Plus "+"	Symbol used to signify addition
Addends	The objects added together
Sum	The result of addition

Example: 5 + 3 = 8
Addend plus addend equals sum

NOTE: It is extremely important for you to be able to quickly add two whole numbers together in your mind. Practice completing the following chart as quickly as you can. It will be more helpful to skip around the chart randomly than to complete an entire row or column at one time. Some entries have been provided for you.

TRY:

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2		3								
3										
4						9				
5										
6									14	
7										
8										
9										

Properties

Commutative property of Addition - The order of two numbers around the addition sign does not affect the sum.	$a+b=b+a$	$5+4=4+5$
Associative property of Addition - The way in which several whole numbers are grouped when they are added, does not affect the final sum.	$(a+b)+c=a+(b+c)$	$(3+4)+5=3+(4+5)$
Identity property of Addition - When 0 is added to any number, the sum is 'identical' to the original number. 0 is the identity element for addition. It is also called the additive identity.	$a+0=0+a=a$	$4+0=0+4=4$

Applying the Associative property of Addition we can group numbers differently to make addition easier.
Add: (4 + 9) + 6 If we write: (4 + 6) + 9 We have 10 + 19 or 19
Addition is often easier if groups of 10 can be formed.

TRY:

7 + 0 = _____ 7 + 4 + 3 = _____

8 + 9 + 2 = _____ 6 + 7 + 2 + 3 + 8 + 4 = _____

When adding larger numbers, be sure to add the digits of the same place value. It the sum of the column is equal to or greater than the place value of the next column, be sure to "carry" to the next column.	6732 + 243 6975	121 *carry* 5397 886 + 6473 12756 Did you group the 10's?

TRY:

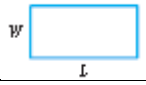
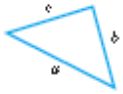
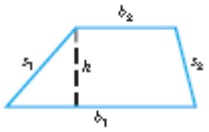
4936	7302
395	303
+ 144	+ 768

Words that Indicate Addition	**Expression**	**Sum**
The sum of seven and four	7 + 4	11
Three is added to fourteen	14 + 3	17
Five more than eight	8 + 5	13
Six increased by two	6 + 2	8
The total of five and nine	5 + 9	14

TRY:

Words that Indicate Addition	**Expression**	**Sum**
The total of six and twenty-one		
Eight more than three		
The sum of sixty and fifty		
Two is added to seventeen		
Five increased by Fifty		

Vocabulary

TYPE	FIGURE	**Perimeter**: The distance around a figure **Perimeter Formulas:**
Rectangle		$P = 2l + 2w$
Square		$P = 4s$
Triangle		$P = a + b + c$
Trapezoid		$P = s_1 + s_2 + b_1 + b_2$

Problems:

1. JC needs to put a wood border around the perimeter of a rectangular living room that is 12 feet wide by 23 feet long. How much wood border will be needed to go around the room?
   Statement: P = 12 + 23 + 12 + 23 Sentence answer: JC needs 70 feet of wood border.

2. Reservations have now closed for the four upcoming shows with 25, 22, 26, and 28 planning to attend. How many total people are expected to attend the shows?
   Statement: T = 25 + 22 + 26 + 28 Sentence answer: A total of 101 people are expected to attend.

TRY:

1. Ti walked around the perimeter of a triangular lot that has sides of 17 yards, 25 yards, and 19 yards. How far did Ti walk?
   Statement: Sentence answer:

2. Five sections of Algebra were offered during the spring with enrollments of 26, 28, 24, 29, and 22. What was the total number of students enrolled in Algebra during the spring?
   Statement: Sentence answer:

Whole Numbers: Subtraction

Vocabulary

Subtraction	The opposite of addition
Minus "-"	Symbol used to signify subtraction
Minuend	The number that something is being subtracted from
Subtrahend	The number being subtracted
Difference	The result of subtraction

Example: 5 - 3 = 2
Minuend minus subtrahend equals difference

The sum of the difference and the subtrahend equals the minuend.

2 + 3 = 5

Difference plus subtrahend equals minuend

TRY the following. Subtract the number down the side from the number across the top.
Do not attempt to complete the boxes that are shaded.

-	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0																				
1																				
2																				
3																				
4																				
5																				
6																				
7																				
8																				
9																				

When subtracting numbers, be sure to subtract the digits of the same place value.

It the value of the digit of the minuend is not large enough, one must borrow 1 from the column to the left.

```
  6789
- 243
  6546
```

borrow

```
  6 1
  7 4
- 3 7
  3 7
```

TRY:

```
  4,976
-   654
```

```
  7,753
-   531
```

TRY:

5, 000	7302
- 654	- 768

Words that Indicate Subtraction	Expression	Difference
Fifteen decreased by seven	15 - 7	8
Eight is subtracted from ten	10 - 8	2
Five less than eight	8 - 5	3
Six less four	6 - 4	2
The difference between nine and five	9 – 5	4

TRY:

Words that Indicate Subtraction	Expression	Difference
Eight is subtracted from thirteen		
Sixteen less eleven		
The difference between nine and two		
Four less than ten		
Ninety-one decreased by seven		

Problems:

1. KT's weekly pay of $ 530 was decreased by $ 75. What amount of pay did KT receive?
   Statement: 530 – 75 = P Sentence answer: KT received $ 455 of pay.

2. A hiker started at an elevation of 784 ft., came down 58 ft., went up 112 ft., and came down 65 ft. What was the hiker's final elevation?
   Statement: 784 – 58 + 112 – 65 = E Sentence answer: The final elevation was 773 ft.

TRY:

1. KT's weekly pay of $ 782 was decreased by $ 63. What amount of pay did KT receive?
   Statement: Sentence answer:

2. A hiker started at an elevation of 635 ft., went up 58 ft., came down 112 ft., and then came down 65 ft. more. What was the hiker's final elevation?
   Statement: Sentence answer:

Whole Numbers: Rounding, Estimation, and Ordering

Vocabulary

Rounding	Expressing numbers to the nearest hundreds, thousand, and so on

Process of Rounding:

Round 17,648 to the nearest 100. Rounded answer: 17,600	1. Look at the digit to the right of the 100's place. (Look at the 4.) 2. Since the digit is less than 5, make it and all other digits to the right zeros.

Round 17,648 to the nearest 1000. Rounded answer: 18,000	1. Look at the digit to the right of the 1000's place. (Look at the 6.) 2. Since the digit is 5 or more, increase the 7 (the 1000's place) by 1 and make the 6 and all other digits to the right zeros.

TRY:

Round each to the …	Nearest thousand	Nearest hundred	Nearest tens
83,238			
149,794			
50,783			

Vocabulary

Estimating	Using rounded numbers to quickly predict an answer

Process of Estimating:

SD would like to have some idea what lunch will cost. Estimate the total bill to the nearest whole dollar.

Item	Original Cost	Estimated Cost
Salad	4.95	5
Soup	2.88	3
Fish	8.25	8
Drink	1.59	2

Estimated cost: E = 5 + 3 + 8 + 2
E = $18

TRY:

Jay is shopping at the local market and is concerned that enough money is available to purchase the items in the cart. Estimate the total bill to the nearest whole dollar.

Item	Original Cost	Estimated Cost
Bread	2.39	
Meat	7.23	
Chips	1.79	
Pop	4.79	

Vocabulary

Inequality signs	Signs used to indicate the relationship of one number or value to another
5 is **less than** 8 written $5 < 8$	-4 is **greater than** -6 written $-4 > -6$
-2 is **less than or equal to** 3 written $-2 \leq 3$	25 is **greater than or equal to** 20+5 written $25 \geq (20+5)$
6 is **not equal to** 5 written $6 \neq 5$	

TRY:

Indicate if the following statements are true or false.

17 < 25 True or False

36 > 39 True or False

(5 + 3) ≤ (3+5) True or False

(14 – 6) ≥ (13 – 2) True or False

Use the < or > symbol to make each statement true.

(5 + 6 – 3) _____ 25

36 _____ (27 – 9 + 10)

17 _____ (25 – 9)

36 _____ (39 – 2)

Whole Numbers: Multiplication

Vocabulary

Multiplication	Repeated addition
X or ·	Symbols used to signify multiplication
Product	The result of multiplication
Factors	The whole numbers multiplied together resulting in a product

Example: 3 · 5 = 15
factor times factor equals product

Multiplying 3 times 5, often written as 3 · 5 or 3 x 5, can be thought of as adding three 5's together.

$3 \cdot 5 = 5 + 5 + 5 = 15$

NOTE: It is just as important for you to be able to quickly multiply in your mind as it is to be able to add. Practice completing the following chart as quickly as you can. Each cell of the table should be the product of the factor at the top of that cell's column and the factor to the far left on that cell's row. It will be more helpful to skip around the chart randomly than to complete an entire row or column. Some entries have been provided for you.

·	0	1	2	3	4	5	6	7	8	9	10	11	12
0													
1													
2				6									
3													
4													
5													
6									48				
7													
8													
9													108
10													
11													
12													

Properties

Commutative property of Multiplication - The order of two numbers around the multiplication sign does not affect the product.	$a \cdot b = b \cdot a$	$5 \cdot 4 = 4 \cdot 5$
Associative property of Multiplication - The way in which several whole numbers are grouped when they are multiplied, does not affect the final product.	$(ab)c = a(bc)$	$(3 \cdot 4) \cdot 5 = 3 \cdot (4 \cdot 5)$

Identity property of Multiplication - When any number is multiplied by 1, the product is 'identical' to the original number. 1 is the identity element for multiplication. It is also called the multiplicative identity.	$a \cdot 1 = 1 \cdot a = a$	$3 \cdot 1 = 1 \cdot 3 = 3$
Multiplicative Property of Zero - When any number is multiplied by 0, the product is 0.	$a \cdot 0 = 0 \cdot a = 0$	$3 \cdot 0 = 0 \cdot 3 = 0$
Distributive property of Multiplication over Addition - To distribute a factor over a sum of two numbers within parentheses, multiply the factor by each number inside the parentheses then add the products. - The Distributive property also works over Subtraction.	$a(b+c) = ab + ac$ $(b+c)a = ba + ca$ $a(b-c) = ab - ac$ $(b-c)a = ba - ca$	$7(5+3) = 7 \cdot 5 + 7 \cdot 3$ $(7+2)3 = 7 \cdot 3 + 2 \cdot 3$ $7(5-3) = 7 \cdot 5 - 7 \cdot 3$ $(7-2)3 = 7 \cdot 3 - 2 \cdot 3$

When multiplying larger numbers, be sure to "carry" to the next column.	Step 1. 6732 X 243 20,196 ← 6,732 * 3 = 20,196 269,280 ← 6,732 * 40 = 269,280 1,346,400 ← 6,732 * 200 = 1,346,400 1,635,876 ← Sum of each part

TRY:

$17 \cdot 1 =$ _____ $0 \cdot 6 =$ _____ $8 \cdot (3 + 5) =$ _____

4936
x 144

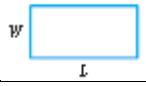
7302
x 768

Words that Indicate Multiplication	Expression	Product
The product of seven and four	$7 \cdot 4$	28
Three times twenty-two	$3 \cdot 22$	66
Twice the sum of sixteen and four	$2 \cdot (16+4)$	40

TRY:

Words that Indicate Multiplication	Expression	Product
The product of nine and eight		
Fifteen times seven		
Twice the sum of nine and six		

Vocabulary

TYPE	FIGURE	**Area**: The number of square units enclosed within the figure **Area Formulas:**
Rectangle		$A = lw$

Problems:

JC needs to put wood flooring down on a rectangular living room that is 12 feet wide by 23 feet long. The flooring costs $7 per square foot. How much wood flooring is needed to complete the room? What will it cost for the wood flooring?

Statement: F = 12 feet · 23 feet

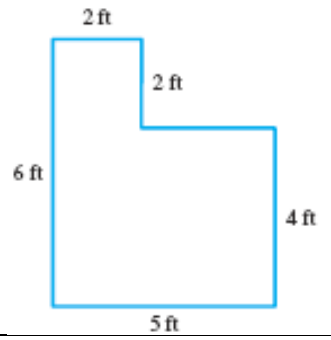
Statement: C = 7 · (12 feet · 23 feet)

C = 7 · 276 ft^2

Sentence answer: JC needs 276 square feet of wood flooring. (also written: 276 ft^2)

It will cost $ 1,932 for the wood flooring.

TRY:

Find the area of the following figure: (Hint: Make two small rectangles. Calculate the area of each rectangle. Then, add the two products together.) If a special metallic adhesive costs $5 per square foot, how much will it cost to cover this figure in with the special metallic adhesive?	

Statement:

Sentence answer:

Whole Numbers: Division

Vocabulary

Division	Repeated subtraction. Dividing two numbers such as 6 divided by 3 can be thought of as asking how many groups of 3 are in the number 6.
Division sign "÷"	Symbol used to signify division
Dividend	The number that is being divided
Divisor	The number used to divide the Dividend
Quotient	The result of subtraction
Remainder	The portion "leftover" when a Dividend is not exactly divisible by the Divisor

Example: $6 \div 3 = 2$ or $6 = 3 \cdot 2$
Dividend divided by Divisor equals Quotient

$7 \div 3 = 2$, remainder 1 or $7 = 3 \cdot 2 + 1$
Dividend divided by Divisor equals Quotient

To check: Divisor · quotient + remainder = Dividend

$6 \div 3 = 2$ check: $3 \cdot 2 + 0 = 6$

$7 \div 3 = 2$, remainder 1 check: $(3 \cdot 2) + 1 = 7$

NOTE:

Division by 0 is not allowed. Consider that $6 \div 3 = 2$ because $3 \cdot 2 = 6$.
Now consider the problem: $6 \div 0$ What number times 0 results in 6? None.
Therefore, **division by 0 is undefined**.
The quotient of $0 \div 6$ is considered to be 0 since $0 \cdot 6 = 0$. 0 divided by any number (except 0) is 0.

TRY:

$17 \div 6 =$ _____ $0 \div 6 =$ _____ $8 \div 0 =$ _____

Long division with remainders:

21 remainder 2 $12\overline{)254}$ 24 14 12 2	TRY: $9\overline{)3525}$

TRY:	
$6\overline{)219}$	$15\overline{)24,005}$

Words that Indicate Division	Expression	Quotient
The quotient of eight and four	8 ÷ 4	2
Twelve divided by three	12 ÷ 3	4

TRY:

Words that Indicate Division	Expression	Quotient
The quotient of ten and two		
Fifteen divided by five		

Problems:

ABC company has bonus money of $5,216 dollars to distribute evenly to the 16 employees. How much bonus will each employee get?

Statement: B = 5216 ÷ 16 Sentence: Each employee will get $326 bonus dollars.

Try:

Ticket receipts for the musical were $268. If each ticket costs $4, how many tickets were sold?

Statement: Sentence:

A child's game requires 14 buttons. How many complete games can be made with 232 buttons?

Statement: Sentence:

Whole Numbers with Exponents

Vocabulary

Base	The number that is the factor in exponential notation. Given 2^3, the 2 is the base.
Exponent	The number of times the base is to be multiplied by itself. Given 2^3, the 3 is the exponent or power.

$2 \cdot 2 \cdot 2$ can be written 2^3 and is read "2 to the third power" $2^3 = 8$

3^2 means $3 \cdot 3$ and is read "3 to the second power" $3^2 = 9$

1^3 means $1 \cdot 1 \cdot 1$ which equals 1 In fact, the number 1 raised to any natural number is 1.

3^0 equals 1 Any whole number , other than 0, raised to the zero power is 1.
(Reasons for this will be explained in a future course.)

TRY evaluating the following expressions as quickly as you can.

$1^0 =$ _____	$2^0 =$ _____	$3^0 =$ _____	$5^0 =$ _____	$7^2 =$ _____
$1^1 =$ _____	$2^1 =$ _____	$3^1 =$ _____	$5^1 =$ _____	$8^2 =$ _____
$1^2 =$ _____	$2^2 =$ _____	$3^2 =$ _____	$5^2 =$ _____	$9^2 =$ _____
$1^3 =$ _____	$2^3 =$ _____	$3^3 =$ _____	$5^3 =$ _____	$10^2 =$ _____
$1^4 =$ _____	$2^4 =$ _____	$4^0 =$ _____	$6^0 =$ _____	$11^2 =$ _____
$1^5 =$ _____	$2^5 =$ _____	$4^1 =$ _____	$6^1 =$ _____	$12^2 =$ _____
$1^6 =$ _____		$4^2 =$ _____	$6^2 =$ _____	$13^2 =$ _____

Powers of 10: Exponential notation with 10 as the base.
Place Value: The place values of our number system correspond to the powers of 10.

Standard (numerical) Form: 5,367
Expanded Form: (5 x 1,000) + (3 x 100) + (6 x 10) + (7 x 1)
Powers of 10 Form: $(5 \times 10^3) + (3 \times 10^2) + (6 \times 10^1) + (7 \times 10^0)$

Standard (numerical) Form: 400,012,508
Expanded Form: (4 x 100,000,000) + (1 x 10,000) + (2 x 1,000) + (5 x 100) + (8 x 1)
Powers of 10 Form: $(4x\ 10^8) + (1 \times 10^4) + (2 \times 10^3) + (5 \times 10^2) + (8 \times 10^0)$

Notice how the exponent on the base 10 corresponds to the number of zeros in the place value.

Using Scientific Notation

Rounding a value to one significant digit (rounding to the first place value on the left of the number) and writing the result using powers of 10 is called using **scientific notation**.

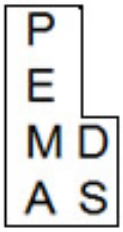
Round 485,352,943 to one significant digit and write the result in scientific notation: $500,000,000 = 5 \times 10^8$

TRY: Round 352,943 to one significant digit and write the result in scientific notation:

Grouping Symbols and Order of Operations

Most mathematical expressions involve using a combination of adding, subtracting, multiplying, dividing or working with exponents. To be sure everyone evaluates these expressions in the same way, certain rules are followed. These rules are called the **Order of Operations**.

1. "P" If parentheses or other grouping symbols such as braces or brackets are present in the expression, evaluate what is in these grouping symbols first.
2. "E" Evaluate all expressions with exponents next.
3. "M, D" Complete any multiplication or division in order, working from left to right.
4. "A, S" Complete any addition or subtraction in order, working from left to right.



A common way to remember this order is to use the phrase: **P**lease **E**xcuse **M**y **D**ear **A**unt **S**uz

Most books use the phrase Aunt Sally – but since my name is Sue and Suz rhymes with Excuse, I use it. Also, remember that Aunt Suz is older and needs a chair to sit on. Therefore, form a chair with the letters to help you remember that multiplication or division are at the same level and that addition or subtraction are at the same level.

Example:

$14 + 10 \div 2 - 3 \cdot 2$	We have no parenthesis or exponents.
$14 + 10 \div 2 - 3 \cdot 2$	The next level is multiplication or division. Since these are on the same level, one does them in order from left to right.
	$10 \div 2 = 5$ divide first, then multiply $3 \cdot 2 = 6$
$14 + 5 - 6$	The next level is addition or subtraction. Again, since these are on the same level, one does them in order from left to right.
	$14 + 5 = 19$ add first
$19 - 6$	then subtract
13	Final answer.

Example:

$2^3 + 6 (12 - 7)$	Evaluate inside the ( ) first: 12 - 7 = 5
$2^3 + 6 (5)$	Evaluate the expression with the exponent: $2^3 = 2 \cdot 2 \cdot 2 = 8$
$8 + 6 (5)$	Complete the multiplication: 6 (5) = 30
	When a number is immediately in front of a bracket, one multiplies that number with what is in the bracket. The multiplication symbol is understood.
8 + 30	Complete the addition: 8 + 30 = 38
38	Final answer.

TRY: $3 (5 - 1) + 5^2 =$

$10 – 3 \{ 4 [3 – (2 – 1)] – 2 (2 + 1) \} =$

$14 - 8 \div 2 \cdot 3 =$

<u>Properly following the rules for Order of Operations is absolutely necessary for being successful with math! Please be sure you understand them and follow them.</u>

Expressions and Equations

Vocabulary

Expression	A number or a meaningful collection of operations (+,-,·,÷) and numbers Examples of expressions: $3 + 7$ $12 - 2$ $8 \cdot 5 \div 4$ $3^2 \div (2+1) + 7$
Evaluate	To determine the value of the expression
Equation	Two expressions connected by an equal sign Examples of equations: $3 + 7 = 12 - 2$ $8 \cdot 5 \div 4 = 3^2 \div (2+1) + 7$
Statement	An equation that can be judged to be either True or False $3 + 7 = 12 - 4$ is a FALSE statement. $3 + 7 = 12 - 2$ is a TRUE statement.

TRY: Identify each of the following as an expression or an equation. (Check the appropriate column.)

	Expression	Equation	If it is an equation, is it True or False?
$2 \cdot 5^3$			
$3^2 \div (2+1) + 7 = 12 - 2$			
$(8 \cdot 5 \div 4)^2 = (5 \cdot 4 \div 2)$			
$(8 \cdot 5 \div 4)^2 + (5 \cdot 4 \div 2)$			

TRY: For each of the following, translate the sentence into an equation then determine if the equation is a True statement or a False statement.

Sentence	Equation	True or False?
Three plus six is two more than seven.		
Four squared less nine is five.		
Five times the sum of one plus three is the same as the product of two and ten.		
The difference of seventeen and thirteen is the quotient of thirty-six and nine.		
Thirty less six is twelve squared.		